Dynamics, Topology and Computations 2022

Book of abstracts

István Balázs

University of Klagenfurt

A differential equation with a state-dependent queueing delay

We consider a differential equation with a state-dependent delay motivated by a queueing process. The time delay is determined by an algebraic equation involving the length of the queue for which a discontinuous differential equation holds. The new type of state-dependent delay raises some problems that are studied in this paper. We formulate an appropriate framework to handle the system, and show that the solutions define a Lipschitz continuous semiflow in the phase space. The second main result guarantees the existence of slowly oscillating periodic solutions.

Jakub Banaśkiewicz

Jagiellonian University

Periodic orbit for the Brusselator system with diffusion.

In the presentation we will discuss some results for the Brusselator system with diffusion which is given by following pde's

$$\begin{cases} u_t = d_1 u_{xx} - (B+1)u + u^2 v + A\sin(x), \\ v_t = d_2 v_{xx} + Bu - u^2 v \ . \end{cases} \tag{1}$$

This sytem can model autocatalytic reactions in form:

$$\begin{aligned} &A \to u, \\ &2u + v \to 3u, \\ &B + u \to v + D, \\ &u \to E. \end{aligned}$$

Coefficients A, B are describing these reactions and coefficients d_1, d_2 are describing rate of diffusion for substances u and v. This is an extension of standard Brusselator which is system of two ordinary differential equations, which for some parameters, has a periodic orbit. It is arising from Hopf bifurcation of stationary point. Numerical experiments shows that periodic solutions should also exists for system with diffusion. We are proving that for some parameters d_1, d_2, A, B the periodic orbit also exists for the system with diffusion and Dirichlet boundary conditions.

This is joint work with Piotr Kalita and Piotr Zgliczyński.

Héctor Barge

Universidad Politécnica de Madrid

Geometric topology and the realization problem of attractors

In this talk we shall be concerned with the realization problem of attractors for homeomorphisms of $\mathbb{R}^3$. More specifically, we shall study under what circumstances a compactum $K \subset \mathbb{R}^3$ can be realized as an attractor for a discrete dynamical systems. We shall see that using techniques from geometric topology it is possible find topological invariants that give us clues for constructing examples of compacta that cannot be so realized. Moreover, we shall see that these invariants are complete in some suitable situations.

Jan Boroński
AGH Kraków

A classification of Lozi maps

Extending a recent work of Crovisier and Pujals on mildly dissipative diffeomorphisms of the plane, we show that Henon-like and Lozi-like maps on their strange attractors are conjugate to natural extensions (a.k.a. shift homeomorphisms on inverse limits) of maps on metric trees with dense set of branch points. In consequence, these trees very well approximate the topology of the attractors, and the maps on them give good models of the dynamics. To the best of our knowledge, these are the first examples of canonical two-parameter families of attractors in the plane for which one is guaranteed such a 1-dimensional locally connected model tying together topology and dynamics of these attractors. For Henon maps this applies to Benedicks-Carleson positive Lebesgue measure parameter set, and sheds more light onto the result of Barge from 1987, who showed that there exist parameter values for which Henon maps on their attractors are not natural extensions of any maps on branched 1-manifolds. For Lozi maps the result applies to an open set of parameters given by Misiurewicz in 1980. Our result can be seen as a generalization to the non-uniformly hyperbolic world of a classical result of Williams from 1967. Relying on this inverse limit description, by a careful analysis of the geometry of Lozi attractors, and combining it with symbolic dynamics model guaranteed by a result of Stimac and Misiurewicz, we provide the following classification: within Misiurewicz parameter set two Lozi maps are conjugate if and only if their sets of kneading sequences coincide, if and only if their folding patterns are the same.

Maxime Breden
Ecole Polytechnique

A posteriori validation of generalized polynomial chaos expansions

Generalized polynomial chaos (gPC) expansions are a powerful tool to study differential equations with random coefficients, allowing in particular to efficiently approximate random invariant sets associated to such equations. In this work, we use ideas from validated numerics in order to obtain rigorous a posteriori error estimates together with existence results about gPC expansions of random invariant sets. This approach also provides a new framework for conducting validated continuation, i.e. for rigorously computing isolated branches of solutions in parameter-dependent systems, which generalizes in a straightforward way to multi-parameter continuation.

Maciej Capiński
AGH University of Science and Technology

A topological version of the normally hyperbolic invariant manifold theorem

We consider perturbations of normally hyperbolic invariant manifolds, under which they can lose their hyperbolic properties. We show that if the perturbed map which drives the dynamical system preserves the properties of topological expansion and contraction, then the manifold is perturbed to an invariant set. The main feature is that our results do not require the ‘rate conditions’ to hold after the perturbation. In this case the manifold can be perturbed to an invariant set, which is not a topological manifold.

Wojciech Chachólski
KTH, Stockholm

Homological algebra and persistence

There is a growing interest in TDA community regarding homological invariants of persistent modules.

In my talk I will describe a set up for relative homological algebra with computationally effective methods based on Koszul complexes for calculating associated Betti diagrams.

This is a join work with A. Guidolin, I. Ren, M. Scolamiero, F. Tombari

Pedro J. Chocano
Rey Juan Carlos University

A first approach to the reconstruction of discrete dynamical systems

Recent results point out that compact metric spaces may be reconstructed using inverse sequences of finite spaces. Then it is natural to set this problem of reconstruction in the context of dynamical systems. Concretely, one may wonder if it is possible to reconstruct a discrete dynamical system by means of a sequence of combinatorial dynamical systems defined on finite spaces. This question leads to first establish a notion of dynamical system for finite spaces. The classical one has been proved to be trivial, so a new one is needed. In this talk, we introduce the notion of Vietoris-like multivalued map and study some of its properties. Later we use it as a first step to approximate some dynamical information of discrete dynamical systems defined on polyhedra. Finally, we will sketch some ideas to construct a new category that can lead to adapt some classical invariants such as the topological degree among others.

Tamal Dey
Purdue University

New Results in Computing Zigzag and Multiparameter Persistence

In this talk we present the following two results: (i) Zigzag persistence, a powerful extension of the standard persistence, is known to be more costly to compute than the non-zigzag persistence. We show how to narrow this efficiency gap. Our main result is that an input zigzag filtration can be converted to a non-zigzag filtration of same length with a very little cost. Furthermore, the barcode of the original filtration can be easily read from the barcode of the converted filtration. Our experiment shows that this indeed allows substantial performance gain over the existing state-of-the-art softwares. We can take advantage of this result for computing generalized rank invariants involving 2-parameter persistence due to our next finding; (ii) We show that the generalized rank over a finite interval I of a 2-parameter persistence module M is equal to the full rank of the zigzag module that is induced on a certain path in I tracing mostly its boundary. Among others, we apply this result to obtain an improved algorithm for determining if a given 2-parameter module is interval decomposable and, if so, to compute all intervals supporting its summands.

Joint work (i) with Tao Hou (ii) with Woojin Kim and Facundo Memoli

Emin Durmishi
University of Tetovo, North Macedonia

Chain Components

The notion of chain connectedness of a set in a topological space is introduced by using open coverings of the space and the notion of chain. Namely, a set $C \subseteq X$ is called chain connected in the topological space X if for any open covering $\mathcal{U}$ of X and any two points $x, y \in C$, there exists a finite sequence of open sets $U_1, U_2, \ldots, U_n$ in $\mathcal{U}$ such that $x \in U_1$, $y \in U_n$ and $U_i \cap U_{i+1} \neq \emptyset$, $\forall i \in \{1, 2, \ldots, n-1\}$.

By a chain component we mean the maximal chain connected set in X. Some properties of chain components are provided. It is shown that chain components and quasicomponents of the space coincide.

Emmanuel Fleurantin
UNC-CH and GMU

A Dynamical Systems Approach for Most Probable Escape Paths in Non-Gradient Systems

Most Probable Escape Paths (MPEPs) have been heavily studied in large deviation theory. Classical methods for computing MPEPs include (but are not limited to) the geometric Minimum Action Method (gMAM) and the string method. In this presentation, we will focus on heteroclinic orbits of a Hamiltonian system derived from the Friedlin-Wentzell action functional with boundary conditions. We compute the unstable manifold of the equilibrium solution and the stable manifold of a periodic orbit which acts as the boundary of the basin of attraction of our base attractor. The MPEPs are then the transversal intersections of those invariant objects. The Maslov index will help us distinguish local minima of the derived Euler-Lagrange equations from all the heteroclinic solutions in our Hamiltonian system. Our computations will then be compared with Monte Carlo simulations in order to understand theoretical predictions. We will use a 2-dimensional autonomous system with a stable fixed point coexisting with a saddle cycle as a focal point for our methodology.

Ernest Fontich
Universitat de Barcelona - CRM

Invariant manifolds of parabolic objects

Parabolic objects such as fixed points, periodic orbits, and invariant tori appear in families of systems, for instance when the objects undergo bifurcations.

Also, in some families, due to the nature of the problem at hand, the parabolic objects exist for all values of some parameter. This happens in some problems of celestial mechanics and chemistry when one studies the behavior near infinity.

In this talk, we will review some results concerning the existence and regularity of invariant manifolds of such objects and discuss some applications.

This is joint work with Inma Baldomá and Pau Martín.

Valery Gaiko
NAS of Belarus

Bifurcation and topological methods for polynomial dynamical systems

We carry out a global qualitative analysis of polynomial dynamical systems. To control limit cycle bifurcations of such systems, especially, bifurcations of multiple limit cycles, it is necessary to know properties and combine effects of all their field rotation parameters. It can be done by means of development of new bifurcation and topological methods based on the Wintner-Perko termination principle for planar polynomial dynamical systems. If we do not know the cyclicity of the termination points, then, applying canonical systems with field rotation parameters, we use geometric properties of the spirals filling the interior and exterior domains of limit cycles. Applying this approach, we have solved, e.g., Hilbert's Sixteenth Problem on the maximum number and distribution of limit cycles for the general Liénard polynomial system with an arbitrary number of singular points, the Kukles cubic-linear system, the Euler-Lagrange-Liénard polynomial mechanical system, Leslie-Gower systems which model the population dynamics in real ecological or biomedical systems, and a reduced planar quartic Topp system which models the dynamics of diabetes. Finally, applying a similar approach, we have considered various applications of three-dimensional polynomial dynamical systems and, in particular, completed the strange attractor bifurcation scenario in Lorenz type systems globally connecting the homoclinic, period-doubling, Andronov-Shilnikov, and period-halving bifurcations of their limit cycles.

Zbigniew Galias
AGH University of Science and Technology

On limit cycles of the Songling system

The Songling system is a three-parameter family of quadratic planar vector fields. This system provides a lower bound on the Hilbert number H_n for $n = 2$ defined in the famous 16th Hilbert problem. We prove that for specific parameter values the Songling system has exactly four limit cycles. Precise bounds for the positions of these limit cycles are given. The proof is carried out using rigorous computational methods based on interval arithmetic. The techniques use are applicable to the much wider class of real-analytic planar differential equations.

Anna Gierzkiewicz
Jagiellonian University

Period forcing for multidimensional maps with attracting periodic orbits

The Sharkovskii Theorem is a powerful tool for proving the existence of periodic orbits and chaotic phenomena in discrete one-dimensional dynamical systems:

Theorem 1 (Sharkovskii) *Define an ordering '$\triangleleft$' of natural numbers:*

$$3 \triangleleft 5 \triangleleft 7 \triangleleft \cdots \triangleleft 2 \cdot 3 \triangleleft 2 \cdot 5 \triangleleft \cdots \triangleleft 2^2 \cdot 3 \triangleleft 2^2 \cdot 5 \triangleleft \cdots \triangleleft 2^k \triangleleft 2^{k-1} \triangleleft \cdots \triangleleft 2^2 \triangleleft 2 \triangleleft 1.$$

Let $f : I \to \mathbb{R}$ be a continuous map of an interval. If f has an n-periodic point and $n \triangleleft m$, then f also has an m-periodic point.

In general, the above Theorem is not valid for multidimensional maps. However, in a joint work with Piotr Zgliczyński, we develop the methods used in [K. Burns, B. Hasselblatt, *The Sharkovsky Theorem: A Natural Direct Proof*, Am Math Mon, 118 No. 3 (2011), 229–244] to prove that Sharkovskii's Theorem can be generalized to the case of higher-dimensional maps with an attracting periodic orbit.

The result is [A. G., P. Zgliczyński, *From the Sharkovskii theorem to periodic orbits for the Rössler system*, J Differ Equ, 314 (2022),733–751]:

Theorem 2 *Consider a continuous map $F : I \times \overline{B}(0, R) \to \operatorname{int}\left(I \times \overline{B}(0, R)\right)$, where $I \subset \mathbb{R}$ is a closed interval and $\overline{B}(0, R) \subset \mathbb{R}^{N-1}$ a closed ball of radius R. Let us denote by (x, y) points in $I \times \overline{B}(0, R)$.*

Suppose that F has an n-periodic point $(x_0, y_0) \in \mathbb{R} \times \mathbb{R}^{N-1}$ with least period n and denote its orbit by $\{(x_0, y_0), (x_1, y_1) = F(x_0, y_0), \ldots, (x_{n-1}, y_{n-1}) = F^{n-1}(x_0, y_0), (x_n, y_n) = (x_0, y_0)\} \subset \operatorname{int} I \times \overline{B}(0, R)$.

Suppose that there exist $\delta_0, \delta_1, \ldots, \delta_{n-1} > 0$ such that

$$\forall\, i \in \{0, \ldots, n-1\} \qquad F\left([x_i \pm \delta_i] \times \overline{B}(0, R)\right) \subset (x_{i+1} \pm \delta_{i+1}) \times B(0, R).$$

Then for every natural number m succeeding n in the Sharkovskii order (1), F has a point with the least period m.

As an application, we prove the existence of n-periodic orbits for almost all $n \in \mathbb{N}$ in the Rössler system, for four sets of parameters. The proof is computer-assisted with the use of CAPD library for C++.

Rodrigo Goncalves Schaefer
Jagiellonian University

Arnold Diffusion for a Hamiltonian system with 3 +1/2 degrees of freedom

In the present work, we study the geometrical mechanism of diffusion in an empha priori unstable Hamiltonian system with $3 + 1/2$ degrees of freedom. This mechanism consists in combining iterates of the emphinner and the emphouter dynamics associated to a emphNormally Hyperbolic Invariant Manifold (NHIM) to build a diffusing emphpseudo-orbit and applying Shadowing results to prove the existence of a diffusing orbit of the system.

Besides proving the existence of diffusion, we concentrate a significant part of our study on a particular family of orbits of the scattering map, the emphHighways, which are enough to ensure a large drift on the action variables and the time of diffusion along them agrees with the optimal estimates in the literature. We proved these properties by applying analytical and numerical approaches. This is a joint work with Amadeu Delshams and Albert Granados.

Alex Haro
Universitat de Barcelona - CRM

Effective bounds for the measure of rotations

A fundamental question in Dynamical Systems is to identify regions of phase/parameter space satisfying a given property (stability, linearization, etc). In this talk, given a family of analytic circle diffeomorphisms depending on a parameter, we obtain effective (almost optimal) lower bounds of the Lebesgue measure of the set of parameters that are conjugated to a rigid rotation. We estimate this measure using an a-posteriori KAM scheme that relies on quantitative conditions that are checkable using computer assistance. We carefully describe how the hypotheses in our theorems are reduced to a finite number of computations, and apply our methodology to the case of the Arnold family. Hence we show that obtaining non-asymptotic lower bounds for the applicability of KAM theorems is a feasible task provided one has an a-posteriori theorem to characterize the problem.

This is joint work with Jordi Lluis Figueras and Alejandro Luque.

David Hien
TU Munich

A TDA approach to cycling in dynamical systems

In the theory of dynamical systems, periodic orbits are of fundamental interest. More generally, there are dynamical phenomena with orbits which are periodic up to a small perturbation, for example quasi-periodic dynamics or orbits in a chaotic attractor. Periodic orbits in a flow can be described topologically as orbits with nontrivial homology in degree 1. Based on this observation, I will introduce the notion of cycling: an orbit is called cycling if an ε-thickening of the orbit has nontrivial homology in degree 1. I will then focus on computational aspects related to cycling which will be illustrated using well-known chaotic attractors.

Olivier Hénot
McGill University
Computer-assisted proofs of radially symmetric steady states for Klein-Gordon on $\mathbb{R}^3$

The talk presents an ongoing work (with J. B. van den Berg, J.-P. Lessard and J. D. Mireles James) on the rigorous computation of radially symmetric steady states for PDEs on unbounded domains.

Specifically, we focus on the 3D Klein-Gordon equation

$$U_t = \Delta U - U + \beta_1 U^2 + \beta_2 U^3, \qquad U := U(t,x) \in \mathbb{R}, t \geq 0, x \in \mathbb{R}^3 \text{ and } \beta_1, \beta_2 \in \mathbb{R},$$

for which we search for solutions of the form $U(x,t) = u(|x|)$ for some function u independent of time. Introducing $r(x) := |x|$, the function u satisfies a second-order ODE on $[0, +\infty)$ with a non-autonomous term r^{-1}.

We show how one devises an operator amenable for a Newton-Kantorovich argument and whose fixed-point yields a solution $r \in [0,\delta] \mapsto u(r)$, for some $\delta > 0$, such that $u(\delta)$ belongs to the stable manifold of 0. In particular, our strategy introduces a centre direction constraining us to study a centre-stable manifold; we discuss how one uses the Lyapunov-Perron operator to obtain its rigorous enclosure.

Jonathan Jaquette
Boston University
Unraveling global dynamics and unstable blowup in a nonlinear Schrödinger equation without conservation laws

Conservation laws and Lyapunov functions are powerful tools for proving the global existence of stability of solutions, but for many complex systems, these tools are often insufficient to understand non-perturbative dynamics. In this talk we discuss the model nonlinear Schrödinger equation $iu_t = u_{xx} + u^2$ with $x \in \mathbb{T} \equiv \mathbb{R}/\mathbb{Z}$, which does not admit any (analytic) conservative or gradient quantities.

In a recent series of papers, together with JP Lessard and A Takayasu, we have used computer assisted proofs to show that this equation exhibits rich dynamical behavior that exist globally in time: non-trivial equilibria, homoclinic orbits, and heteroclinic orbits, and integrable subsystems foliated by periodic orbits. I will discuss these results, and current work toward understanding unstable blowup and wave turbulence.

Oliver Junge
Technical University of Munich
Entropic transfer operators

We propose a new concept for the regularization and discretization of transfer operators in dynamical systems. Our approach is based on the entropically regularized optimal transport between two probability measures. In particular, we use optimal transport plans in order to construct a finite-dimensional approximation of some transfer operator which can be analyzed computationally. We prove that the spectrum of the discretized operator converges to the one of the regularized original operator, give an analysis of the relation between the discretized and the original peripheral spectrum for a rotation map on the n-torus and provide code for three numerical experiments, including one based on the raw trajectory data of a small biomolecule from which its dominant conformations are recovered.

Piotr Kalita
Jagiellonian University

Autonomous and non-autonomous unbounded attractors in evolutionary problems

If the semigroup is slowly non-dissipative, i.e., its trajectories can diverge to infinity as time tends to infinity, one still can study its dynamics via the approach by the unbounded attractors - the counterpart of the classical notion of global attractors. We continue the development of this theory started by Chepyzhov and Goritskii in [V.V. Chepyzhov and A.Yu. Goritskii. Unbounded attractors of evolution equations, volume 10 of Advances in Soviet Mathematics, pages 85–128. American Mathematical Society, 1992]. We provide the abstract results on the unbouded attractor existence, and we study the properties of these attractors, as well as of unbounded ω-limit sets in slowly non-dissipative setting. We also develop the pullback non-autonomous counterpart of the unbounded attractor theory. The abstract theory that we develop is illustrated by the analysis of the autonomous problem governed by the equation $u_t = Au + f(u)$. In particular, using the inertial manifold approach, we provide the criteria under which the unbounded attractor coincides with the graph of the Lipschitz function, or becomes close to the graph of the Lipschitz function for large argument.

Shu Kanazawa
Kyoto University

Large deviation principle for persistence diagrams of random cubical filtrations

A random cubical filtration is an increasing family of random cubical sets, which are the union of randomly generated higher-dimensional unit cubes with integer coordinates. The objective of this work is to investigate the asymptotic behavior of the (random) persistence diagrams of a random cubical filtration model as the window size tends to infinity. Recently, the strong law of large numbers for the persistence diagrams was proved by Hiraoka, Miyanaga, and Tsunoda, which states that the persistence diagram converges vaguely to a deterministic measure almost surely.

In this talk, we are interested in the decay rate of the probability that the persistence diagram is far from the deterministic limiting measure. We first show the large deviation principles for Betti numbers, persistent Betti numbers, and the histograms generated by counting the birth-death pairs falling in each fine rectangular region. The key tool for the proofs is a general large deviation principle for regular nearly additive processes established by Seppäläinen and Yukich. Furthermore, we develop a method of lifting a large deviation principle for persistence histograms to persistence diagrams by the exponentially good approximation approach.

This talk is based on joint work with Yasuaki Hiraoka, Jun Miyanaga, and Kenkichi Tsunoda.

Shane Kepley
VU Amsterdam

Efficient parameterization of invariant manifolds using deep neural networks

Spectral methods are the gold standard for parameterizing manifolds of solutions for ODEs because of their high precision and amenability to computer assisted proofs. However, these methods suffer from several drawbacks. In particular, the parameterizations are costly to compute and time-stepping is far more complicated than other methods.

In this talk we demonstrate how computing these parameterizations and accurately time-stepping can be reduced to a related manifold learning problem. The latter problem is solved by training a deep neural network to interpolate charts for a low dimensional manifold embedded in a high dimensional Euclidean space. This training is highly parallelizable and need only be performed once. Once the neural network is trained, it is capable of parameterizing invariant manifolds for the ODE and time-stepping with remarkable efficiency and precision.

Bernd Krauskopf
University of Auckland

The structure of accumulating global bifurcations of two coupled phase-amplitude oscillators

We consider a four-dimensional vector field that arises as the semiclassical approximation of two coupled, driven and lossy photonic crystal nanocavities — optical devices that operate with only a few hundred photons. Mathematically, this system describes the dynamics of two coupled phase-amplitude oscillators that are driven with strength f and frequency detuning δ. We employ advanced tools from bifurcation theory, in combincation with the computation of kneading invariants and of maximum Lyapunov exponents, to determine the bifurcation diagram in the plane of (f, δ)-plane. Its complicated structure of accumulating global bifurcations is organised by a number of codimension-two bifurcations, including a homoclinic flip, a Bykov T-point and new types of degenerate singular heteroclinic cycles. In particular, we identify in this way regions with different types of chaotic switching between the two oscillators.

This is joint work with Andrus Giraldo (KIAS) and Neil Broderick (Auckland)

Karolina Lademann
University of Gdansk

Effective highly accurate integrators for linear Klein-Gordon equations from low to high frequency regimes

In this talk we will introduce an efficient class of numerical schemes for the Klein–Gordon equation which are highly accurate for a variety of oscillations, ranging from small to huge. The construction of the methods is based on well-known Magnus expansion. However, in the case of the Klein-Gordon equation, the order of convergence of the methods differs from the theory known in the literature.

More precisely, Magnus expansion components do not scale only in terms of time step Δt, but also in terms of the oscillation frequencies ω_n. Therefore, the final error of the numerical methods can be represented as

$$\min\left\{h^3, \frac{h^2}{\omega_{\min}}, h^5\omega_{\max}^2\right\}.$$

Several numerical experiments highlight our theoretical findings and underline the efficiency of the new schemes.

Resuts are obtained in collaboration with Karolina Kropielnicka (IM PAN) and Katharina Schratz (Sorbonne Université).

Jean-Philippe Lessard
McGill University

Towards computational Morse-Floer homology: forcing results for connecting orbits by computing relative indices of critical points

To make progress towards better computability of Morse-Floer homology, and thus enhance the applicability of Floer theory, it is essential to have tools to determine the relative index of equilibria. Since even the existence of nontrivial stationary points is often difficult to accomplish, extracting their index information is usually out of reach. In this talk, we present a computer-assisted proof approach to determining relative indices of stationary states. Moreover, we show how forcing results can be then used to prove theorems about connecting orbits and traveling waves in PDEs. We demonstrate how these ideas work in practice for the Cauchy-Riemann equations, a travelling wave problem on the square and the Ohta-Kawasaki equation.

This is joint work with Jan Bouwe van den Berg, Marcio Gameiro and Rob Vandervorst.

Michał Lipiński
Dioscuri Centre in TDA, IMPAN

Tracking Dynamical Features via Continuation and Persistence

In recent years, combinatorial dynamics have become an important subject of interest due to their potential in computational methods. Forman's combinatorial vector fields theory became a cornerstone of combinatorial models for continuous-time dynamical systems. Recently, Mrozek extended Forman's idea by proposing a much more flexible theory of multivector fields. The theory was enriched with the combinatorial theory of the Conley index and other mathematical objects allowing an extensive study of combinatorial systems. In the following works, a prominent topological data analysis tool, the persistent homology, has been used to study the robustness of combinatorial isolated invariant sets or to track changes in the Conley index and Morse-Conley graph.

Our current work focuses on the idea of continuation. Two isolated invariant sets are said to be related by a continuation if one can be transformed into the other by a continuous deformation of a dynamical system. In particular, the Conley index of the isolated invariant set stays intact throughout the transformation.

Our first goal was to adapt the concept of continuation into combinatorial settings of multivector fields. To this end, we had to introduce a combinatorial counterpart of a continuous deformation of a system. Due to the finite realm of our settings, the natural choice for the minimal perturbation of a system is an atomic refinement, i.e., splitting a multivector into two smaller multivectors. This imposes a topology on the space of all multivector fields and facilitates the construction resembling the classical definition of continuation.

Secondly, we propose a tracking protocol, a canonical way of studying the evolution of an isolated invariant set. With the method, one can follow a chosen isolated invariant set, observe how it travels in phase space, and get hints where the set is passing through a bifurcation (at least on the level given by a data or a resolution). Moreover, we present the construction in the spirit of persistent homology. In particular, we show that the continuation is a special case of the persistence of the Conley index.

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Ugo Locatelli
Mathematics Dept. - Univ. Rome Tor Vergata

Computer assisted proofs of existence of KAM tori: a normal form approach

In a recently published article (L. Valvo, U.L.: *Hamiltonian control of magnetic field lines: Computer assisted results proving the existence of KAM barriers*, J. Comp. Dyn., 2022), we prove the existence of quasi-periodic motions in a quite challenging problem because the integrable approximation is subject to a quite large perturbation. All the codes that are needed to perform the Computer Assisted Proof (hereafter, CAP) are collected in a software package that is publicly available from the Mendeley Data repository. These codes are designed in such a a way to be rather easy-to-use, also for what concerns eventual adaptations for applications to different problems.

The main goal of this talk concerns with the description of the few ideas, which are really essential to understand the working mechanism of this software package for doing CAPs of existence of KAM tori. Particular attention will be devoted to the extensions to quasi-integrable Hamiltonians with more than two degrees of freedom and infinite expansions in Fourier series.

Jason Mireles James
Florida Atlantic University

Continuation and Bifurcation of Ejection-Collision Orbits

Ejection-collision orbits in the planar circular restricted three body problem describe the motion of particles which collide with one primary in finite forward time, and with another primary in finite backward time (that is, they are ejected). In this talk I will focus on the case where the ejection-collision involves both primaries.

Recently the speaker, with Maciej Capinski and Shane Kepley, developed a computer assisted method of proof for establishing the existence of such orbits at a fixed value of the Jacobi constant and for a fixed mass ratio. In this talk I will discuss additional computer assisted techniques for proving the existence of continuous branches and bifurcations of ejection-collision orbits. I will also discuss the large velocity limiting case for these orbits and present some numerical results concerning the properties of the shortest ejection/collision as parameters are varied. This is joint work with Giani Arioli.

Konstantin Mischaikow
Rutgers Univerisity

Solving Systems of Ordinary Differential Equations via Combinatorial Homological Algebra

TBA

Irina Mitrea
Temple University

Computational Aspects for Elliptic Boundary Value Problems in Non-Smooth Domains

Elliptic boundary value problems in non-smooth domains with Dirichlet and Neumann type boundary conditions arise naturally in connection with physical phenomena such as conductivity, heat transfer, elastic deformations, and electrostatics.

In this talk I will survey well-posedness results for second-order elliptic boundary problems in two dimensions. These results are obtained through a blend of Calderon-Zygmund theory methods, Mellin transform techniques, and validated numerics. This work is part of an ongoing collaboration project with Warwick Tucker.

Małgorzata Moczurad
Jagiellonian University

Central configurations on the plane with N heavy and k light bodies

We study the problem of planar central configurations with N heavy bodies and k bodies with arbitrary small masses. We derive the equation which describe the limit of light masses going to zero, which can be seen as the equation for central configurations in the anisotropic plane. Using computer rigorous computations we compute all central configurations for $N = 2$ and $k = 3, 4$ and for the derived limit problem. We show that the results are consistent.

This is a joint work with Piotr Zgliczyński.

David Mosquera-Lois
Universidade de Santiago de Compostela

Homotopic distance and dynamics

Let $f, g\colon X \to Y$ be two continuous maps between topological spaces. We introduce a homotopic invariant, that we call homotopic distance between f and g, denoted $\mathrm{D}(f, g)$, which measures how far are the maps from being homotopic. It generalizes the Lusternik–Schnirelmann category and the topological complexity. Moreover, we prove a result relating the dynamics on a manifold with the homotopic distance between any two maps which have such manifold as domain. This generalizes the Lusternik-Schnirelmann theorem (for Morse functions), and a similar result by Farber for the topological complexity. This is joint work with E. Macías-Virgós and M.J. Pereira-Sáez.

Hinke Osinga
University of Auckland

Heterodimensional cycles as organising centres of complicated dynamics

A heterodimensional cycle consists of a pair of heteroclinic connections between two saddle periodic orbits with unstable manifolds of different dimensions. We study heterodimensional cycles in a four-dimensional vector field, where such cycles are characterised by a connecting orbit that lies in the intersection of two two-dimensional manifolds; the return connection is given by a family of connecting orbits in the structurally stable two-dimensional intersection of two three-dimensional manifolds. Heterodimensional cycles are known to organise highly complicated dynamics, which persist under C^1-perturbations of the vector field. We employ continuation techniques on a two-point boundary value problem set-up of Lin's method to compute heterodimensional cycles and associated nearby global bifurcations. We find that our four-dimensional vector field exhibits cycles of four different types that are distinguished by the orientability properties of the tangent bundles associated with the periodic orbits. In this talk, we present transitions from one cycle type to another and the effect such changes in orientability have geometrically. We explore how heterodimensional cycles contribute to the organisation of the overall bifurcation structure, which in turn, elucidates mechanisms behind the generation of C^1-robust chaotic dynamics.

Paweł Pilarczyk
Gdańsk University of Technology

How much stochastic dynamics is there in the quadratic map family?

We study the quadratic family of one-dimensional maps $f_a(x) = a - x^2$. We conduct comprehensive numerical analysis of collections of finite orbits of the critical point, computed for *intervals of parameter values* using rigorous numerical methods. We use the computer to explicitly construct a collection of several thousand parameter intervals, contained in $\Omega = [1.4, 2]$, that are *proved* to have a specific so-called *escape time*, which roughly means that some effectively computed iterate of the critical point taken over all the parameters in that interval has considerable width in the phase space. In particular, we compute a *rigorous lower bound* on this width, in addition to the upper bound. We investigate the effect of certain constraints imposed on the numerical computations upon the resulting collection of intervals. Additionally, we illustrate and discuss the distribution of the computed intervals in the parameter space. The purpose of our work is to establish grounds for further numerical computation of a lower bound on the measure of stochastic parameters in Ω. The source code of the software and the data discussed in the talk is freely available at `http://www.pawelpilarczyk.com/quadr/`. A web page interface is also provided that allows carrying out some limited computations. The ideas and procedures introduced in the talk can be easily generalised to apply to other parametrized families of dynamical systems. This is joint work with Stefano Luzzatto (Abdus Salam International Centre for Theoretical Physics, Trieste, Italy).

Mateusz Przybylski
Jagiellonian University

The Szymczak functor on the category of finite relations

The Szymczak functor is a tool used to construct Conley index for discrete-time dynamical systems. Due to a certain key property, it enables the correct definition of the index. Moreover, the functor is universal in the sense that any other functor with this property factorizes through the Szymczak functor. The universality of the Szymczak functor shows its generality, but is also responsible for its computational weakness, because there is no general method to tell whether two objects in the Szymczak category (i.e. target category of the functor) are isomorphic or not.

In this talk, I will present an algorithmizable classification of isomorphism classes in the Szymczak category over the category of finite sets with arbitrary relations as morphisms. Such a classification may provide a new method to study multivalued dynamical systems represented by relations. These multivalued dynamical systems arise naturally from a dynamics given by the data, that is, a sampled dynamics.

This is a joint work with Marian Mrozek. Based on: M. Mrozek, M. Przybylski, The Szymczak Functor on the Category of Finite Sets and Finite Relations, arXiv:2203.08525 [math.DS], preprint.

Elena Queirolo
TUM

Validation in Machine Learning

Recurrent Neural Networks have been a workhorse of Machine Learning, being the main architecture for a variety of purposes, such as classification and language processing. But the reasons behind their success is often unclear, and further clouded by the shear size of the systems under consideration. With the recent introduction of NeuralODEs, it is possible to relate RNNs to traditional ODEs. We can apply tools from dynamical systems to tackle problems related to the stability and behaviour of RNNs. In this talk, new results are presented to give insight on the dynamics supported by RNNs. In particular, local bifurcations are detected in a broad class of RNNs, and initial results on chaotic behaviour are discussed.

Damian Sadowski
Jagiellonian University

Computational approach to dynamics based on combinatorial multivector fields.

Joint work with Donald Woukeng Feudjio, Jakub Leśkiewicz and Marian Mrozek

For a given smooth 2-D vector field, or a finite sample of 2-D vectors, and a triangulation of a compact subset of the phase space, we construct a combinatorial multivector field citeaa. In the case of a given smooth 2-D vector field our construction, based on the transversality, allows us to study dynamics induced by the vector field in a rigorous way. This in turn can lead to a computer-assisted proof of, for example, an existence of periodic solutions citebb. In the case of sampled vectors this provides insight into the dynamics of the underlying system. The presentation will be oriented on sampled dynamics. I will talk about the construction of a combinatorial multivector field from a cloud of 2-D vectors. I will show examples (Van der Pol oscillator) when such a construction leads to good insight into dynamics.

References

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[2] Marian Mrozek, Roman Srzednicki, Justin Thorpe, and Thomas Wanner. "Combinatorial vs. classical dynamics: Recurrence". In: Communications in Nonlinear Science and Numerical Simulation 108 (2022), pp. 106–226.

Evelyn Sander
George Mason University

Rigorous bifurcation methods for diblock and triblock copolymer models

The Ohta-Kawasaki copolymer model exhibits a rich equilibrium bifurcation structure. In this talk, we describe computer-assisted proof methods computer-assisted proof technique for bounding the norm of the inverses of certain fourth-order elliptic operators, in combination with an application of a constructive version of the implicit function theorem. We illustrate these methods by showing how they can be used to validate equilibrium solutions in triblock copolymers, as well as how to validate symmetry breaking pitchfork bifurcations in diblock copolymers for solutions with cyclic symmetry higher than $\mathbb{Z}_2$.

This work is joint with Thomas Wanner and Peter Rizzi.

Jakub Tomaszewski
Jagiellonian University

The relations between disjointness and common factors for distal systems

It is well known that if two dynamical systems are disjoint, then they have no common factors, but the converse implication is not true. However, Berg has shown that for a measurable distal system and ergodic system those conditions are equivalent. It required to show that distal and ergodic systems are quasi-disjoint. It has been shown by Joel Moreira, Florian K. Richter and Donald Robertson that distal systems are quasi disjoint from any measure preserving system. During the talk we will examine the analogous statements for topological dynamical systems, showing that topological distal systems are quasi-disjoint from any other system and thus for those having no common factors implies topological disjointness.

Warwick Tucker
Monash University

Lower bounds on the Hausdorff dimensions of Julia sets

We present an algorithm for a rigorous computation of lower bounds on the Hausdorff dimensions of Julia sets for a wide class of holomorphic maps. We apply this algorithm to obtain lower bounds on the Hausdorff dimension of the Julia sets of some infinitely renormalizable real quadratic polynomials, including the Feigenbaum polynomial $p_{Feig}(z) = z^2 + c_{Feig}$. In addition to that, we construct a piecewise constant function on $[-2, 2]$ that provides rigorous lower bounds for the Hausdorff dimension of the Julia sets of all quadratic polynomials $p_c(z) = z^2 + c$ with $c \in [-2, 2]$. Finally, we verify the conjecture of Ludwik Jaksztas and Michel Zinsmeister that the Hausdorff dimension of the Julia set of a quadratic polynomial $p_c(z) = z^2 + c$, is a C^1-smooth function of the real parameter c on the interval $c \in (c_{Feig}, -3/4)$.

This is joint work with Artem Dudko and Igors Gorbovickis.

Jan Bouwe van den Berg
VU Amsterdam

Computer assisted proofs for spiral waves in the complex Ginzburg-Landau problem

The cubic complex Ginzburg-Landau equation is among the prototypical systems in the study of pattern formation. One of its dynamic features is the occurence of rotating spirals. Due to special symmetries of this partial differential equation, a spiral wave Ansatz reduces the problem to a nonautonomous ordinary differential equation. Finding spiral waves thus corresponds to establishing certain connecting orbits in a finite dimensional dynamical system. In this talk we discuss how to approach this problem via computer assisted proof techniques. We use a domain decomposition which allows a combination of Taylor series, Chebyshev series and the parametrization of a center-stable manifold.

Thomas Wanner
George Mason University

Combinatorial Topological Dynamics

Morse theory establishes a celebrated link between classical gradient dynamics and the topology of the underlying phase space. It provided the motivation for two independent developments. On the one hand, Conley's theory of isolated invariant sets and Morse decompositions, which is a generalization of Morse theory, is able to encode the global dynamics of general dynamical systems using topological information. On the other hand, Forman's discrete Morse theory on simplicial complexes, which is a combinatorial version of the classical theory, and has found numerous applications in mathematics, computer science, and applied sciences. In this talk, we introduce recent work on combinatorial topological dynamics, which combines both of the above theories and leads as a special case to a dynamical Conley theory for Forman vector fields, and more general, for multivectors. This theory has been developed using the general framework of finite topological spaces, which contain simplicial complexes as a special case.

Daniel Wilczak
Jagiellonian University

Hyperbolic horsheshoe for Kuramoto-Sivashinkii equation

We discuss a computer assisted proof for the existence of a hyperbolic set with horshoe-like behavior (symbolic dynamics) for Kuramoto-Sivashinskii PDE on the line for the parameter values just past of the sequence of period doubling bifurcations.

Natalia Wodka-Cholewa
AGH UST

Computer assisted proof of diffusion - application to PER3BP

Joint work with Maciej Capiński

In this talk we consider the Planar Elliptic Restricted Three Body Problem (PER3BP), which describes the motion of a massless body in gravitational influence of two large bodies (primaries). The elliptic problem is treated as a perturbation of the circular problem (PCR3BP) and the perturbation parameter ε is the eccentricity of the primaries. In [1] we present a computer assisted proof of diffusion of the considered problem. We show that for sufficiently small perturbations we have orbits with explicit energy changes. The change does not depend on the size of the perturbation. The result is based on existence of trajectories that shadow transversal intersections of stable and unstable manifolds.

References

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Donald Woukeng Feudjio
Jagiellonian University

Rigorous computation in dynamics based on combinatorial multivector fields

A computational study of dynamical systems given explicitly by a formula or only via a finite sample requires combinatorial tool. Among such tools is the concept of combinatorial multivector field, an extension of the Forman's concept of combinatorial vector field which may be studied by algorithmic means. The construction of a combinatorial multivector field combined with transversality may lead to computer-assisted proofs. However, the construction itself is a challenge that we intend to address. In particular, the construction of a multivector enclosing a stationary point, if not taken care of by special means, may lead to a loss of information at the end of the computation of our combinatorial multivector field, due to some properties that multivectors must satisfy. During the talk we will focus on dynamical system given explicitly by a formula. Hence we will first introduce an algorithm for the construction of a transversal polytope around each stationary point of some 2-D dynamical systems. Then, we will introduce an algorithm constructing combinatorial multivector fields using rigorous computation, from a 2-D vector field derived from an autonomous differential equation on a triangulation of a compact subset of $\mathbb{R}^2$, with certain transversality relation with respect to the flow. We will proceed with the computation of Morse sets to show the features we can extract from our systems using those algorithms. The transversal polytope will be necessary while computing the Morse sets since it will allow us to easily separate all the stationary points with other features such as periodic orbits during the computation of Morse sets. This will lead us to computer-assisted proof of the existence of periodic orbits in some 2-D dynamical systems. Joint work with: Damian Sadowski, Jakub Leskiewicz and Marian Mrozek

Piotr Zgliczyński
Jagiellonian University

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