Arai, Zin (Creative Research Institute "Sousei", Hokkaido University N21W10 Kita-ku, Sapporo, 001-0021, Japan) *Development and Applications of an Algorithm for Proving Structural Stability*
In this talk, we discuss an algorithm for verifying the structural stability of dynamical systems. It is based on algorithms for computing rigours outer approximations of global objects such as maximal invariant sets and chain recurrent sets, and the fact that uniform hyperbolicity is equivalent to quasi-hyperbolicity under the assumption of chain-recurrence. To handle higher dimensional and more difficult problems, we also introduce an implementation of a succinct data structure for reducing memory usage. Applications of the algorithm include the computation of monodromy actions of complex dynamical systems and the pruning front of real planner dynamical systems, and the verification of the non-degeneracy of bifurcations.

Ashwin, Peter (Mathematics Research Institute School of Engineering, Computing and Mathematics University of Exeter Exeter EX4 4QF, UK) *Boundedness of orbits for cone exchange transformations*
This talk will discuss some joint work with Prof A Goetz (San Francisco State University) on a class of planar piecewise isometric systems: cone exchange transformations. They provide a natural way to extend interval exchange dynamics of an angular variable to a planar context. This class of system includes a number of interesting types of area preserving dynamical systems, from dual polygonal billiards through to certain signal processing systems. A particular result we discuss is the role of the average radial motion of points near infinity. More precisely we should that ergodic properties of the interval exchange at infinity can be related to boundedness of trajectories for such systems.
This is joint work with A. Goetz.

Barari, Amin (Department of Civil Engineering, University of Mazandaran, Iran) *Analysis of Blasius Equation for Flat-Plate Flow with Infinite Boundary Value*
This paper applies the homotopy perturbation method (HPM) to determine the well-known Blasius equation with infinite boundary value for Flat-Plate Flow. We study here the possibility of reducing the momentum and continuity equations to ordinary differential equations by a similarity transformation and write the nonlinear differential equation in the state space format, and then solve the initial value problem instead of boundary value problem. The significance of linear part is a key factor in convergence. A first seen linear part may lead to an unstable solution, therefore an extra term is added to the linear part and deduced from the nonlinear section. The results reveals that HPM is very effective, convenient and quite accurate to both linear and nonlinear problems. It is predicted that HPM can be widely applied in engineering. Some plots and numerical results are presented to show the reliability and simplicity of the method.

Barakat, Mohamed (Fachrichtung Mathematik, Universität des Saarlandes, 66041 Saarbrücken, Germany) *Experiments with the package* `conley`
This talk will introduce the software package `conley` which was developed in Aachen during the last few years. Starting from some topological data stemming from a Morse decomposition `conley` is able to compute all possible connection (and transition) matrices. Beside the well-known ways of establishing the existence of connecting orbits, the talk will present a different way emphasizing the

importance of computer assisted proofs.
This is joint work with Stanislaus Maier - Paape.

Berz, Martin (Department of Physics and Astronomy 3253 BPS, MSU) *Rigorous High-Order Enclosures of Manifolds, Homoclinic Points, and Symbolic Dynamics*
We discuss high order methods to obtain tight C0 and C1 enclosures of stable and unstable manifolds based on Taylor models. Using novel tools for iteration of discrete and continuous systems including dynamic domain decomposition, we provide enclosures of stable and unstable manifolds of very extended lengths. Using a Taylor model based rigorous global optimizer, all homoclinic points of a collection of manifold are found and their genealogic hierarchy is established. Using this information, automatic methods are developed to untangle the resulting trellis and determine bounds for the associated topological entropy. The methods are illustrated in the plane for the case of the Henon map, where collections of several thousand homoclinic points are determined and new sharp lower bounds for entropy are found.
This is joint work with Kyoko Makino, Johannes Grote ND Sheldon Newhouse.

Capiński, Maciej (AGH University of Science and Technology, Krakow, Poland) *Finding Normally Hyperbolic Invariant Manifolds Around L1 in the RC3BP - computer assisted proof*
The libration point L1 in the spatial circular restricted three body problem is of a saddle x center x center type. This means that by the normally hyperbolic invariant manifold theorem, in the vicinity of this point, there exists an invariant manifold of the system. Such manifolds have been extensively investigated numerically and made use of for space mission design. An open question remains however on how to prove their existence at a given distance away from L1. We will show a topological method which enables us to prove that such manifolds exist within a given investigated region. The method is tailored for rigorous computer assisted implementation. Some preliminary computer assisted results for the Earth-Sun system will be presented.
This is joint work with Pablo Roldan.

Celletti, Alessandra (Department of Mathematics University of Roma Tor Vergata) *On the dynamics of nearly-integrable, dissipative systems*
We present some results on the dynamics of nearly-integrable systems which are subject to a dissipation. These systems are ruled by two parameters: the perturbing parameter, measuring the strength of the nonlinearity, and the dissipative parameter. The existence of invariant attractors is investigated through KAM theory and their non-existence through an extension of converse KAM theory. We also provide numerical methods to compute the break-down threshold of the invariant attractors as a function of the perturbing and dissipative parameters. Close to this limit we study the cantori, providing a proof of the existence of dissipative cantori in the case of the sawtooth map.

Collins, Pieter (Centrum Wiskunde en Informatica Postbus 94079, 1090 GB Amsterdam The Netherlands)*Computation of Reachable Sets of Hybrid Systems*
In this talk we consider the problem of computing reachable sets of nonlinear hybrid systems. We first discuss the semantics of evolution required to make the reachable sets computable. We then show how the computations are imple-

mented in the tool Ariadne.

Dłotko, Paweł (Uniwersytet Jagielloński, Kraków, Poland) *Computational homology and cohomology theory in the electromagnetism*
In recent years new formulations and efficient algorithms in the computational electromagnetism have been developed. These algorithms work on the simplicial triangulation that meshes the considered physical object and provide fast and high quality solutions of Maxwell's equations. However, these algorithms require some prior knowledge of the topology of a mesh of the considered physical object. In the presentation the techniques used so far by the electrical engineers to obtain this topological information will be briefly described and the limitations of these techniques will be highlighted. Then it will be demonstrated that the topological information needed are the first cohomology group generators. At the end some recent developments of the algorithms and (co)homology computations code will be presented.

Ethier, Marc (Département de mathématiques Université de Sherbrooke 2500, boul. de l'Université Sherbrooke (Québec), Canada J1K 2R1), *Analysis of Singular Zones in Multidimensional Discrete Data*
In this talk, we propose a homological method inspired by the Conley index theory for the detection and classification of singularities in discrete multidimensional scalar fields. Our method makes use of the fact that critical points of a function f can be grouped into connected components C. These components may be regarded as invariant sets of the flow associated to the gradient field ∇f. If the components are bounded and isolated, the Conley index theory can be applied to analyse their criticality, using as isolating neighborhoods an extension of the *wraps* defined in Allili *et al.* (2007). We improve the results of that paper and present experimentation with 3D data fields, where visual techniques solely based on counting connected components of super-level and sub-level neighborhoods are insufficient.
This is joint work with Madjid Allili and Tomasz Kaczynski.

Ferrario, Davide (University of Milano Bicocca Department of Mathematics and Applications Via R. Cozzi, 53 20125 Milano) *Variational and topological properties of n-body minimizers*
Periodic orbits of the n-body problem can be found as minimizers under suitable symmetry and topological constraints. Existence of collisionless orbits and qualitative properties of minimizers can be studied and visualized by combining theoretical, computational and symbolic algebraic techniques. Both the theoretical and the computational side of the problem will be illustrated.

Frosini, Patrizio (Dipartimento di Matematica Piazza di Porta San Donato, 5 40126 Bologna ITALIA) *Recent advances in multidimensional persistent topology*
Persistent Topology allows to study properties that can be described by means of $\mathbb{R}^k$-valued functions defined on topological spaces, with respect to changes of these functions. The possible applications range from quantifying the difference between Riemannian structures on the same manifold to comparing coloured surfaces for purposes of pattern recognition. Indeed, in both cases we have to compute how much a morphism can preserve the $\mathbb{R}^k$-valued function describing the considered property (the Riemannian structure or the colour, in our two examples).

While the case $k = 1$ has been studied by many researchers, very little is known about the multidimensional case ($k > 1$). In this talk we describe some new results that have been recently proved for multidimensional Persistent Topology. They are based on a new method, reducing the problem to the 1-dimensional case by a suitable foliation. This approach has led to the proof of stability of multidimensional persistent homology groups and to the localization of their discontinuities when 0-homology is involved, opening the way to new computational methods and applications.

Denis Gaidashev (University of Uppsala) *Dynamics of the Universal Area-Preserving Map Associated with Period Doubling*

It is known that the famous Feigenbaum-Coullet-Tresser period doubling universality has a counterpart for area-preserving maps of $\mathbb{R}^2$. A renormalization approach has been used by Eckmann, Koch and Wittwer in a computer-assisted proof of existence of a *"universal"* area-preserving map — a map with orbits of all binary periods $2^k, k \in \mathbb{N}$. We consider maps in some neighbourhood of this map and study their dynamics.

We present a mixture of analytic and rigorous computational results pertaining to the universal map and *"infinitely renormalizable"* maps; specifically, existence of a *bi-infinite heteroclinic tangle* and, for any $N \in \mathbb{N}$, an N-component horseshoe homeomorphic to a topological Markov chain on the space of all two-sided sequences composed of N symbols, as well as existence of *unbounded* and *oscillating* orbits. We will also describe certain hyperbolic sets for such maps, and show how distortion tools can be used to compute their Hausdorff dimension.

For all locally infinitely renormalizable maps we prove the existence of an invariant *"stable"* set $\mathcal{C}_F^\infty$ such that the maximal Lyapunov exponent of $F|_{\mathcal{C}_F^\infty}$ is zero. We address the issues of rigidity of this set for infinitely renormalizable maps, and obtain bounds on its Hausdorff dimension.
This project is a joint work with Tomas Johnson, Uppsala University.

Galante, Joseph (Department of Mathematics Mathematics Building University of Maryland College Park, MD 20742-4015 USA)*Destruction of High Eccentricity Invariant Curves Through Comparison of Action*
We consider the restricted circular planar three body problem with Sun-Jupiter mass ratio μ and Jacobi constant C denoted $RCP3BP(\mu, C)$. For $\mu = 0.001$ and $C = 1.8$ there are three Hill Regions. We restrict to dynamics the unbounded "Outer Hill Region".

Sample Result 1 *The* $RPC3BP(0.001, 1.8)$ *possesses an orbit* $A(t) = (x, y, x', y')(t)$ *whose osculating eccentricity in the past is approximately equal to 0.75 and A(t) escapes to infinity with small positive speed in the future; more precisely* $|e(A(t)) - 0.75| \leq 0.01$ *for* $t < 0$ *and* $e(A(t)) > 1 + \rho$ *for some* $\rho > 0$ *and all large* t*. Moreover,* $A(t)$ *never comes closer then* 1.6 *to the origin.*

We outline how one can go about proving the sample result using Aubry-Mather theory and Mather's variational method, and how to use rigorous integration to obtain the lower bound of 0.75.
Joint work with Vadim Kaloshin

Galias, Zbigniew (Department of Electrical Engineering AGH University of Science and Technology al. Mickiewicza 30 30-059 Kraków, Poland) *Rigorous*

results on short periodic orbits for the Lorenz system
We study rigorously the problem of existence of periodic orbits for the Lorenz system by means of a symbolic dynamics approach combined with interval methods. Symbolic dynamics is used to find approximate initial positions of periodic points and interval operators are used to prove the existence of periodic orbits in a neighborhood of the computer generated solution. We consider the Lorenz system:

$$\dot{x}_1 = sx_2 - sx_1, \quad \dot{x}_2 = rx_1 - x_2 - x_1x_3, \quad \dot{x}_3 = x_1x_2 - qx_3,$$

where $s = 10$, $r = 28$, $q = 8/3$. The Poincaré map is defined by the hyperplane $\Sigma = \{x = (x_1, x_2, x_3) \colon x_3 = 27, \dot{x}_3 < 0\}$. Let γ denotes the first intersection of the stable manifold of the origin with the return plane Σ. Each trajectory is labelled in the following way: if the trajectory intersects Σ to the left of γ, then the intersection point is labelled with L, otherwise it is labelled with R. In order to study periodic orbits we consider periodic symbol sequences $s = (s_0, s_1, \ldots, s_{n-1})$, where $s_k \in \{\mathrm{L}, \mathrm{R}\}$, for $k = 0, 1, \ldots n-1$. It can be shown that a periodic symbol sequence corresponds to at most one periodic orbit.

To find all short periodic orbits a trajectory of P composed of 10^6 points was generated. We have considered all periodic symbol sequences with the principal period $n \leq 14$. We have shown that both symbol sequences with the principal period $n = 1$ are not admissible and that every other sequence corresponds to exactly one periodic orbit of P. We have confirmed that there are exactly 2536 periodic orbits of P with the period $n \leq 14$, which is in agreement with non-rigorous results (compare [D. Viswanath, *Symbolic dynamics and periodic orbits of the Lorenz attractor*, Nonlinearity. 16 (2003), 1035–1056]).
This is joint work with Warwick Tucker.

Gidea, Marian (Department of Mathematics Northeastern Illinois University 5500 N. St. Louis Avenue Chicago, IL 60625 USA) *A shadowing lemma for normally hyperbolic invariant manifolds and applications to the Arnold diffusion problem*
We consider a normally hyperbolic invariant manifold for a smooth map, where the stable and unstable manifolds intersect transversally. There exist outer dynamics on the normally hyperbolic invariant manifold defined along the heteroclinic excursions, and there also exist inner dynamics defined by the restriction of the map to the normally hyperbolic invariant manifold. We show the following shadowing lemma type of result: given a bi-infinite sequence of topological rectangles in the normally hyperbolic invariant manifold, where the successive pairs of rectangles are correctly aligned by the outer dynamics alternatively with the inner dynamics, then there is a trajectory that visits some small neighborhoods of the rectangles in the prescribed order. We apply this shadowing lemma to a model for the Arnold diffusion problem. In this case, the normally hyperbolic invariant manifold is diffeomorphic to an annulus, and the inner dynamics is given by a twist map. We show how to link together trajectories along the heteroclinic excursions with trajectories connecting the boundaries of Birkhoff Zones of Instability or Aubry-Mather sets in the annulus, in order to obtain trajectories that travel far in the phase space.

Gierzkiewicz, Anna (Institute of Mathematics, Jagiellonian University, Łojasiewicza 6, 30-348 Kraków, Poland), *Chaotic dynamics in isolating segments*

Jaulin, Luc (DTN(Laboratoire Développement des Technologies Nouvelles) ENSIETA (Ecole Nationale Supérieure des Ingénieurs des Etudes et Techniques d'Armement), 2 rue Fran cois Verny, 29806 Brest Cédex 09, France) *Interval methods with applications to robotics*
Interval analysis makes it possible to solve a large class on nonlinear problems such as

i computing all global minimisers of a nonconvex criterion,

ii computing all solutions of a set of nonlinear equations,

iii characterizing sets defined by nonlinear inequalities, ...

Unlike classical numerical approaches (Monte Carlo or local methods, for instance), the results provided by interval analysis are obtained in a guaranteed way and in a finite time, even when strong nonlinearities and discontinuities are involved in the problem. Combined with constraint propagation techniques, interval methods make possible to deal efficiently with high dimensional problems (with more than 1000 variables for instance). The purpose of this presentation is to introduce in a pedagogical way the principles of interval methods and constraint propagation techniques. Some applications to robotics (such as path-planning, dynamic localization, viability analysis) will be given.

Johnson, Tomas (Uppsala University Department of Mathematics P.O. Box 480 SE-751 06 Uppsala Sweden) *Constructing planar vector fields with many limit cycles*
An accurate method to compute enclosures of Abelian integrals is developed. This allows for an accurate description of the phase portraits of planar polynomial systems that are perturbations of Hamiltonian systems. As an application of the method we construct a new lower bound on the value of the upper bound of the number of limit cycles that can bifurcate from a Hamiltonian vector field of degree five, when it is perturbed by a polynomial perturbation of degree five; the new bound is 27.

Juda, Mateusz (The Faculty of Mathematics and Computer Science, Jagiellonian University, prof. Stanisława Łojasiewicza 6, 30-348 Kraków) *$\mathbb{Z}^2$-homology of p-manifolds may be computed in $O(n)$ time*
In a short presentation bottom-up approach for computing $\mathbb{Z}_2$ homology groups of 2-manifolds will be presented. Main result is an algorithm with complexity $O(nlog^*n)$ which for a 2-manifold X computes $H_1(X,\mathbb{Z}_2)$ and $H_2(X,\mathbb{Z}_2)$. The algorithm could be useful as an extension for the coreduction homology algorithm.

Koch, Hans (Department of Mathematics 1 University Station C1200 Austin, TX 78712-0257 USA) *Non-Smooth Invariant Tori for Analytic Hamiltonians, and Computer-Assisted Proofs*
Our analysis focuses on the nature of invariant tori in Hamiltonian systems with two degrees of freedom. Based on a large number of numerical studies, it is believed that (among other things) there exists a "critical manifold" in a space of such Hamiltonians, characterized by the existence of a continuous but non-smooth invariant torus. We will describe a computer-assisted proof for the existence of a finite codimension surface of this type. This includes a discussion

of various issues that come up when trying to reduce such a problem to something that is manageable by a computer.

Koch, Hans (Department of Mathematics 1 University Station C1200 Austin, TX 78712-0257 USA) *Computer-assisted methods for the study of dissipative PDEs*
The general context of our analysis are evolution equations in Banach spaces of analytic functions. On the computer, analytic functions are represented by (Fourier) polynomials, with interval coefficients, plus bound on unknown "higher order" terms. We will describe two types of results, both for the unidimensional Kuramoto-Sivashinski (KS) equation. The first is a partial description of the bifurcation diagram for stationary solution of the KS equation, involving 23 bifurcations and 44 branches. The second line of work implements estimates on the flow, its derivative, and associated Poincare maps. The aim (work in progress) is to control long orbits via the method of correctly aligned windows. This is joint work with Gianni Arioli.

Kosiorowski, Grzegorz (Institute of Mathematics, Jagiellonian University, Łojasiewicza 6, 30-348 Kraków, Poland) *Detecting periodic orbits: guiding functions and periodic segments*

Kulczycki, Marcin (Institute of Mathematics, Jagiellonian University. Łojasiewicza 6, 30-348 Kraków, Poland) *AASP - a new kind of average shadowing*
The notion of Asymptotic Average Shadowing Property, a kind of limit shadowing with the average error of orbits tending to zero, has been introduced by Gu in 2007 and is already the subject of several papers. It will be illustrated how this simply defined property can be surprisingly hard to verify even for very uncomplicated examples of maps.

Kułaga, Tomasz (Institute of Mathematics, Jagiellonian University Łojasiewicza 6, 30-348 Krakow, Poland) *C++ application for hyperbolicity verification*
We present an application written in C++ developed to rigorously prove semi-hyperbolicity of $\mathcal{C}^1$-diffeomorphism invariant set. From the semi-hyperbolic theory it follows that for these functions semi-hyperbolicity of the invariant set implies its hyperbolicity. The program is now applicable for the R^2 case for the Hénon like mappings where the computations can be performed either for single or interval parameter values. We give some technical details on how the program is written, what kind of tools and libraries are used and present brief explanation of the crucial steps of the algorithms.

Lessard, Jean - Philippe (Rutgers University Department of Mathematics 110 Frelinghuysen Road Piscataway, NJ, 08854 USA) *Rigorous Computation of Smooth Branches of Periodic Solutions of Delay Equations*
Periodic solutions are objects of fundamental importance in the study of nonlinear functional delay equations. A wide range of analytic and topological tools (e.g. global bifurcation theorems, Fuller index, Conley index theory and equivariant degree theory) have been used and developed to prove results concerning their existence. However, in practice, these tools are extremely difficult to apply. In this talk, we introduce a computational approach that combines a priori analytic estimates, classical numerical analysis techniques and the uniform contraction principle, in order to compute rigorously global branches of

periodic solutions of delay equations. We end the talk by showing how to use this method to partially answer a fifty years old conjecture concerning the slowly oscillating periodic solutions of the famous Wright's equation.

Makino, Kyoko (Department of Physics and Astronomy 3253 BPS, MSU)
High-Order Verified Flow Integrators based on Taylor Models
We present recent enhancements to the COSY-VI rigorous verified integrator for flows of ODEs. Some of these features are dynamic domain decomposition, enhancements of step size control, the use of error variables, and rigorous high-order Poincare sections. We will show applications from restricted three body dynamics, in particular the motion of near-Earth asteroids. For the first time, it is possible to provide verified integration of the dynamics of the Apophis asteroid through its two close approaches with Earth in the next decades. We also show results of integration of flows and determination of Poincare sections for the Lorenz system.

Maier - Paape, Stanislaus(Institut für Mathematik RWTH Aachen)
(Re-)definition of connection matrices
Although the concept of connection matrices naturally leads to braids as already proposed in [Franzosa,1989] we show, based on the work of [Barakat and Roberts, 2009], that exactly the same concept is obtained by only imposing isomorphisms of long exact sequences. Clearly this makes the verification of connection matrices a lot easier.
This is joint work with Mohamed Barakat.

Mireles - James, Jason (Department of Mathematics The University of Texas at Austin, 1 University Station, C1200 Austin, Texas 78712 Phone: (512) 471-7711 Fax: (512) 471-9038)*Computation of Heteroclinic Branched Manifolds by Parameterization*
Let $f : \mathbb{R}^3 \to \mathbb{R}^3$ be a diffeomorphism with hyperbolic fixed points p_1 and p_2. Suppose that $\dim(W^s(p_1)) = \dim(W^u(p_2)) = 2$ and that $\Omega \equiv W^s(p_1) \cap W^u(p_2)$ is transverse and nonempty. If $x \in \Omega$ then x is heteroclinic from p_2 to p_1. Further, by transversality there is a one-dimensional curve $\gamma : (-\tau, \tau) \subset \mathbb{R} \to \mathbb{R}^3$ so that $\gamma(0) = x$ and $\gamma \subset \Omega$. Then the collection of heteroclinic orbits from p_2 to p_1 is a one-dimensional (possibly branched) manifold. We give a numerical scheme for computing branches of Ω, based on the parameterization method for the two dimensional stable and unstable manifolds at p_1 and p_2. Once parameterizations of the stable and unstable manifolds are determined, the heteroclinic connections are expressed as solutions of algebraic equations. These equations are solved and continued using a codimension one Newton Method. The method is illustrated explicitly for the volume preserving Henon family of maps.
This is joint work with Hector Lomeli.

Osajda, Damian (Instytut Matematyczny Uniwersytet Wrocławski pl. Grunwaldzki 2/4 50-384 Wrocław, Polska) *Simplicial non-positive curvature*
I will define systolic (i.e. simplicially non-positively curved) complexes as done (independently) by Chepoi, Haglund and Januszkiewicz-Świątkowski. I will show some interesting properties of those complexes and of groups acting on them geometrically. Several applications of the theory will be explained. I will also describe our latest attempts towards generalization of the notion of simplicial non-positive curvature.

Oprocha, Piotr (Departamento de Matemáticas, Universidad de Murcia, Campus de Espinardo, 30100 Murcia, Spain) *Chaos and semiconjugacy arguments*
The usual technique to prove that the dynamics of a given system is chaotic is to construct a semiconjugacy with another chaotic system (e.g. the full shift on two or more symbols, a shift of finite type, etc.). In this talk we will survey common topological notions of chaos together with sufficient conditions on semiconjugacy which allow to transfer them from the factor to its extension (which is a subset of our system). We will also highlight problems that appear in this approach.

Petrov, Nicola (Department of Mathematics University of Oklahoma 601 Elm Avenue, PHSC 802 Norman, OK 73019 USA)*Principle of Approximate Combination of Scaling Exponents*
This is a joint work with Rafael de la Llave (University of Texas, Austin, USA) and Arturo Olvera (UNAM, Mexico City, Mexico).

We propose a new principle, called the *Principle of Approximate Combination of Scaling Exponents* (*PACSE* for short) that interprets certain relations between scaling exponents in terms of simple semi-global properties of the renormalinzation group for some dynamical systems.

Many transitions to chaotic behavior have been found to have scaling relations related to universal exponents which in turn are related to the combinatorics of the transition. Popular examples are the exponents describing the the accumulation of the kneading sequences in unimodal maps (in particular, the accumulation of period doubling bifurcations), the accumulation of phase lockings in critical circle maps, the scaling phenomena in the dynamics on the boundary of Siegel disks, the breakdown of critical invariant circles of area-preserving twist maps. In each case, the scaling exponents are universal in the sense that they depend on the phenomena considered but not on the actual family exhibiting the phenomena. The existence of universal scaling exponents has been explained as a manifestation of the properties of an appropriately defined renormalization operators.

For simplicity, here we consider only the case of critical circle maps. Consider the two-parameter family of circle maps $f_{\omega,\beta} : \mathbb{T}^1 \to \mathbb{T}^1$ defined by

$$f_{\omega,\beta}(x) = [x + \omega + \beta g(x)] \bmod 1 \ , \tag{1}$$

where g is a smooth periodic function of period 1. If $f'(x)$ becomes 0 at one point c, we say that f is a *critical map* and call c the *critical point* of f. For $a_j \in \mathbb{N}$, let $\langle a_1, a_2, \ldots\rangle$ stand for the number with continued fraction expansion $1/(a_1 + 1/(a_2 + \cdots))$. Let $\mathsf{A} = (a_1, \ldots, a_p)$ and $\mathsf{B} = (b_1, \ldots, b_q)$ be finite sequences of natural numbers, $\mathsf{AB} = (a_1, \ldots, a_p, b_1, \ldots, b_q)$ be their concatenation, and for $n \in \mathbb{N}$ define $\mathsf{A}^{n+1} = \mathsf{AA}^n$. Let $\langle \mathsf{AB}^n\rangle$ stand for the number with continued fraction expansion

$$\langle \mathsf{AB}^n\rangle = \langle a_1, \ldots, a_p, \underbrace{b_1, \ldots, b_q, b_1, \ldots, b_q, \ldots, b_1, \ldots, b_q}_{\text{the sequence } \mathsf{B} \text{ repeated } n \text{ times}}\rangle \ .$$

There are two types of scaling exponents:

(a) *Parameter-space scaling.* For a fixed value of β in (**??**), let $I_n(\beta)$ be the *phase locking interval*, i.e., the interval of values of the parameter ω for which $f_{\omega,\beta}$ has rational rotation number $\frac{P_n}{Q_n} = \langle \mathsf{AB}^n \rangle$. The lengths of the phase locking intervals behave with n as

$$|I_n(\beta)| \approx C\delta_{\mathsf{B}}^{-n} ,$$

where δ_{B} is a universal number that depends only on B and on the order of the critical point c, but not on the particular family of maps (when the families range over a small enough neighborhood).

(b) *Configuration-space scaling (scaling of recurrences).* If f be a critical circle maps with critical point c and rotation number $\langle \mathsf{AB}^\infty \rangle$, then the iterates $f^{Q_n}(c)$ approach c as follows:

$$\left| f^{Q_n}(c) - c \right| \approx C\alpha_{\mathsf{B}}^{-n} ,$$

where α_{B} is a universal number (in the same sense as for δ_{B}).

For circle maps PACSE can be expressed as follows:

(i) For a fixed order of criticality of c, there exist constants C_1 and C_2 s.t.

$$C_1 \leq \frac{\delta_{\mathsf{AB}}}{\delta_{\mathsf{A}}\delta_{\mathsf{B}}} \leq C_2 , \qquad C_1 \leq \frac{\alpha_{\mathsf{AB}}}{\alpha_{\mathsf{A}}\delta_{\mathsf{B}}} \leq C_2 ,$$

where C_1 and C_2 depend only on $\max(a_1, \ldots, a_p, b_1, \ldots, b_q)$.

(ii) For a fixed order of criticality of c and fixed A and B, the following limits exist:

$$\lim_{k\to\infty} \frac{\delta_{\mathsf{A}^k\mathsf{B}}}{(\delta_{\mathsf{A}})^k \delta_{\mathsf{B}}} , \qquad \lim_{k\to\infty} \frac{\alpha_{\mathsf{A}^k\mathsf{B}}}{(\alpha_{\mathsf{A}})^k \alpha_{\mathsf{B}}} .$$

(iii) For a fixed order of criticality of c and fixed A and B, the ratios $\mathcal{D}_k = \frac{\delta_{\mathsf{A}^k\mathsf{B}}}{(\delta_{\mathsf{A}})^k \delta_{\mathsf{B}}}$ and $\mathcal{A}_k = \frac{\alpha_{\mathsf{A}^k\mathsf{B}}}{(\alpha_{\mathsf{A}})^k \alpha_{\mathsf{B}}}$ approach their limits $\mathcal{D}_\infty$ and $\mathcal{A}_\infty$ exponentially:

$$|\mathcal{D}_k - \mathcal{D}_\infty| \approx C\xi^k , \qquad |\mathcal{A}_k - \mathcal{A}_\infty| \approx C\eta^k ,$$

for some constants ξ and η.

We give some numerical values in Table **??**. To appreciate the fact that $\mathcal{D}_k$ and $\mathcal{A}_k$ have limits as $k \to \infty$, note how large the values of the δ's are.

We have performed similar computations related to the dynamics of iterates on the boundary of Siegel disks, and on invariant circles of area-preserving twist maps, as well as for unimodal maps of the interval (for which the operation analogous to concatenation of continued fraction expansions is the so-called $*$-operation among kneading sequences).

We propose a conjectural explanation of the our numerical observations based on the geometry of the function space related to the action of the corresponding renormalization operators. Below we the briefly describe our conjecture, leaving out all technical details.

For each "combinatorics" A (i.e., a continued fraction expansion or a kneading sequence), there is a corresponding renormalization operator $\mathcal{R}_{\mathsf{A}}$, and let p_{A} be a fixed point of $\mathcal{R}_{\mathsf{A}}$. We assume that p_{A} is hyperbolic, and that $W^{\mathrm{s}}_{\mathsf{A}}$ and $W^{\mathrm{u}}_{\mathsf{A}}$ are the stable and unstable manifolds of p_{A}; we assume that the unstable manifolds are one-dimensional. Let B be another such combinatorics, and $\mathcal{R}_{\mathsf{B}}$, p_{B}, $W^{\mathrm{s}}_{\mathsf{B}}$ and $W^{\mathrm{u}}_{\mathsf{B}}$ be the corresponding objects. We assume that $W^{\mathrm{u}}_{\mathsf{A}}$ intersects $W^{\mathrm{s}}_{\mathsf{B}}$

Table 1: Values of the scaling constants δ_{1^k2} and α_{1^k2} and the ratios $\mathcal{D}_k$ and $\mathcal{A}_k$ for $k = 1, \ldots, 11$. In the calculations we used the values $\delta_1 = 2.8336106559$, $\alpha_1 = 1.28857456$, $\delta_2 = 6.79922516$, $\alpha_2 = 1.58682670$.

k	δ_{1^k2}	α_{1^k2}	$\mathcal{D}_k = \dfrac{\delta_{1^k2}}{(\delta_1)^k\delta_2}$	$\mathcal{A}_k = \dfrac{\alpha_{1^k2}}{(\alpha_1)^k\alpha_2}$
1	17.66905276	1.9691355	0.9170936095	0.96302277
2	52.04449	2.590589	0.953118	0.9832182
3	145.425152	3.308635	0.940068655	0.9745199
4	414.51561	4.28301	0.945628	0.978997
5	1171.7123	5.5067	0.94332356	0.97682
6	3323.73	7.1039	0.944333	0.97793
7	9413.7	9.14860	0.94389	0.977369
8	26681	11.7923	0.94411	0.977671
9	75590	15.1929	0.9439	0.977519
10	214000	19.579	0.943	0.97761
11	607900	25.230	0.945	0.97765

transversally, and let $W^{\mathrm{u}}_{\mathsf{A}} \cap W^{\mathrm{s}}_{\mathsf{B}} = \{h_{\mathsf{AB}}\}$; similarly, assume that $W^{\mathrm{u}}_{\mathsf{B}}$ intersects $W^{\mathrm{s}}_{\mathsf{A}}$ transversally, and $W^{\mathrm{u}}_{\mathsf{B}} \cap W^{\mathrm{s}}_{\mathsf{A}} = \{h_{\mathsf{BA}}\}$.

We relate this construction to the existence and properties of a fixed point of the renormalization operator $\mathcal{R}_{\mathsf{A}^m\mathsf{B}^n}$ related to the combined combinatorics $\mathsf{A}^m\mathsf{B}^n$. To this end, we construct a pseudoorbit of the renormalization operators $\mathcal{R}_{\mathsf{A}}$ and $\mathcal{R}_{\mathsf{B}}$ that consists of forward and backward iterates of the point h_{BA} under $\mathcal{R}_{\mathsf{A}}$ and forward and backward iterates of h_{AB} under $\mathcal{R}_{\mathsf{B}}$. Roughly speaking, it starts at h_{BA}, then approaches p_{A} along $W^{\mathrm{s}}_{\mathsf{A}}$ (by forward iterations of $\mathcal{R}_{\mathsf{A}}$), then (while near p_{A}) jumps to $W^{\mathrm{u}}_{\mathsf{A}}$ and moves toward h_{AB} (by backward iterates of $\mathcal{R}_{\mathsf{A}}$); when it comes close to h_{AB}, it jumps to h_{AB} and then moves along $W^{\mathrm{s}}_{\mathsf{B}}$ towards p_{B} (by forward iterations of $\mathcal{R}_{\mathsf{B}}$), then similarly jumps to $W^{\mathrm{u}}_{\mathsf{B}}$ and follows it to a point near h_{BA} (by backward iterations of $\mathcal{R}_{\mathsf{B}}$); finally, once it comes close to h_{BA}, it jumps to h_{BA}, thus closing the periodic orbit. In doing this, we arrange that the pseudoorbit spends approximately m iterations near p_{A} and approximately n iterations near p_{B}, so that it "picks up" a factor of $(\delta_{\mathsf{A}})^m$ and a factor of $(\delta_{\mathsf{B}})^n$ (these are the expanding eigenvalues of the linearizations of the operators $\mathcal{R}_{\mathsf{A}}$ and $\mathcal{R}_{\mathsf{B}}$ near their corresponding fixed points p_{A} and p_{B}). Under some quite reasonable assumptions, one can prove the existence of a true orbit of the combined actions of $\mathcal{R}_{\mathsf{A}}$ and $\mathcal{R}_{\mathsf{B}}$, or of a periodic point of the renormalization operator $\mathcal{R}_{\mathsf{A}^m\mathsf{B}^n}$ corresponding to the combined combinatorics $\mathsf{A}^m\mathsf{B}^n$.

This explains the fact that the ratio of the parameter-space scaling exponents, $\frac{\delta_{\mathsf{A}^k\mathsf{B}}}{(\delta_{\mathsf{A}})^k\delta_{\mathsf{B}}}$, tend to a limit as k tends to ∞ (see part (ii) of PACSE). The other parts of PACSE can also be explained within this construction.
This is joint work with Rafael de la Llave and Arturo Olvera.

Piękoś, Łukasz (K. Gumiński Department of Theoretical Chemistry Faculty of Chemistry, Jagiellonian University, R. Ingardena 3 30-060 Kraków, Poland)
Modeling chemical reactions using molecular dynamics
In this talk we present results of applications of molecular dynamics to model-

ing of chemical reactions. Molecular systems under consideration include non-bridged half-metallocene titanium complexes with aryloxo ligand.

Methodology within Born-Oppeheimer approximation includes Born-Oppenheimer molecular dynamics on the semiempirical quantum chemistry level and Car-Parinello molecular dynamics on the DFT (density functional theory) level.

Presented results include simulations in non-zero temperature, which allows us to take into account entropic effects. Free molecular dynamics is used to study conformational stability of molecular systems under consideration. Since chemical reactions often involve overcoming significant energy barriers they are rare events. In such cases we use constrained molecular dynamics in slow-growth approach, in which constraint value is being changed at sufficiently low rate so the molecular system remains approximately equilibrated.

Pilarczyk, Paweł (Universidade do Minho Centro de Matemática Campus de Gualtar 4710-057 Braga Portugal) *Finite resolution dynamics based on open covers*

We develop a new mathematical model for describing a dynamical system at limited resolution (or finite scale), and we give precise meaning to the notion of a dynamical system having some property at finite resolution. Open covers are used to approximate the topology of the phase space in a finite way, and the dynamical system is represented by means of a combinatorial multivalued map. We translate notions of transitivity and mixing known for general dynamical systems into the finite setting in a consistent way. Moreover, we formulate equivalent conditions for these properties in terms of graphs, and provide effective algorithms for their verification. As an application we show that the Hénon attractor is topologically mixing at all resolutions coarser than 10^{-5}.

A finite family $\mathcal{U}$ of open subsets of a bounded metric space (X, d) such that $X = \bigcup \mathcal{U}$ is called a *cover* of X. We use the elements of $\mathcal{U}$ as a finite approximation of the topology on X. We assume that the cover $\mathcal{U}$ is *essential*, that is, there exists $\varepsilon > 0$ such that every $U \in \mathcal{U}$ contains some point $x \in X$ such that $B(x, \varepsilon) \subset U$ and $B(x, \varepsilon) \cap W = \emptyset$ for all $W \in \mathcal{U} \setminus \{U\}$.

We define the *outer resolution* $\mathcal{R}^+(\mathcal{U})$ of a cover $\mathcal{U}$ as the maximal diameter of its elements. The *inner resolution* $\mathcal{R}^-(\mathcal{U})$ of a cover $\mathcal{U}$ is the supremum of the numbers $d > 0$ such that every ball $B(x, d) \subset X$ is contained in some $U \in \mathcal{U}$, or 0 if such $d > 0$ does not exist.

We represent a map $f \colon X \to X$ by means of a *combinatorial map*$\mathcal{F} \colon \mathcal{U} \multimap \mathcal{U}$ on a cover $\mathcal{U}$ of X, which is a multi-valued (or set-valued) map $\mathcal{F} \colon \mathcal{U} \ni U \mapsto \mathcal{F}(U) \subset \mathcal{U}$. We say that the combinatorial map $\mathcal{F}$ is a *representation* of f if for every $U \in \mathcal{U}$ the set $\mathcal{F}(U)$ contains $\{W \in \mathcal{U} : W \cap f(U) \neq \emptyset\}$.

Let P be a property defined in general for a dynamical system, and let $\mathcal{P}$ be its counterpart defined for a combinatorial map. We say that a map $f \colon X \to X$ *satisfies P at all resolutions* $> \varepsilon$ if every representation $\mathcal{F} \colon \mathcal{U} \multimap \mathcal{U}$ of f such that $\mathcal{R}^-(\mathcal{U}) > \varepsilon$ satisfies $\mathcal{P}$.

We translate the notions of transitivity and mixing known for general maps into the finite-resolution setting in a consistent way, and we provide explicit algorithms for the verification of these properties at the combinatorial level.

As an example of applicability of our theory, we study the Hénon map $H_{a,b}(x, y) = (1 + y - ax^2, bx)$ for the so-called "classical" parameter values $a = 1.4$, $b = 0.3$, as defined in the original Hénon's paper. For the Hénon attractor we prove that there exists an open set $X \subset \mathbb{R}^2$ such that $H_{a,b}(X) \subset X$ and $H_{a,b}|_{X \to X}$ is mixing at all resolutions $> 10^{-5}$, for all (a, b) in some open

set $P \subset \mathbb{R}^2$ containing the pair $(1.4, 0.3)$.
This is joint work with Steffano Luzzatto.

Prokopenya Alexander (The College of Finance and Management Sokolowska 172, 08-110 Siedlce, Poland)*On Stability of Equilibrium Solutions in the Restricted Many-Body Problems*
In this talk we consider the problem of stability of the equilibrium solutions in the Newtonian circular restricted four-body problem. We indicate the difficulties arising in studying this problem and demonstrate application of methods of the KAM-theory for their solving. Using the computer algebra system Mathematica, we construct the Birkhoff's canonical transformation, reducing the Hamiltonian function to the normal form up to the fourth order. As a result we have proved theorems on the Liapunov stability of equilibrium positions in the planar case and their stability for the majority of initial conditions in the spatial case. Besides, we have investigated an influence of the third- and the fourth-order resonances on the stability of equilibrium solutions and proved that only the third-order resonance $\omega_1 = 2\omega_2$ gives rise to their instability.
This is joint work with Leszek Gadomski.

Roldan, Pablo (Departament de Matemàtica Aplicada I. Universitat Politècnica de Catalunya. ETSEIB, Av. Diagonal, 647, 08028 Barcelona. Spain) *Arnold's mechanism of diffusion in the spatial circular restricted three-body problem: a semi-numerical argument.*

Francis Sergeraert (Institut Fourier, Université Grenoble, BP 74, 38402 Saint-Martin-d'Heres Cedex, France) *Algorithms for Topological Invariants*
The standard methods "computing" the various topological invariants frequently meet intermediate objects which are NOT of finite type, even when the original studied object is perfectly finite; this raises hard theoretical and concrete computability problems. Several general methods have been designed to overcome this essential difficulty and at this time only one has been concretely implemented as a computer program, a method called EFFECTIVE HOMOLOGY. We present the main ideas which are used in this method, the nature of the results so obtained and a small typical computer demo. Note also that if one can successfully handle infinite objects, one can also process finite but huge objects such as digital images with a large number of pixels; this point will also be briefly considered.

Simó, Carles (Dept. de Matemàtica Aplicada i Anàlisi, Universitat de Barcelona Gran Via, 585 08007 Barcelona) *Obstructions to integrability of Hamiltonian systems using high order variational equations*
The topic of this work are non-integrability criteria, based on differential Galois theory and requiring the use of higher order variational equations. After recalling the basic theoretical results [**?**, **?**, **?**] a general methodology is presented to deal with these problems with algebraic/analytic tools, as introduced in [**?**]. Several examples will be presented to illustrate the methods, like a family of Hamiltonian systems which require the use of order k variational equations, for arbitrary values of k, to prove non-integrability. Using third order variational equations we prove the non-integrability of a non-linear spring-pendulum problem for the values of the parameter for which the lack of integrability cannot be decided by using first order variational equations. In a similar way the degener-

ate cases of the Swinging Atwood's Machine are studied [**?**]. For quite general problems the required analytical estimates can be unfeasible. Then a numerical method is introduced to give strong evidence of non-integrability, by using jet transport procedures along arbitrary complex paths to arbitrary order [**?**].
This is joint work with Regina Martínez.

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Strelcyn, Jean - Marie (Laboratoire de Mathématiques Raphaël Salem UMR 6085 CNRS Departement de Mathématiques Université de Rouen, France) *Isochronicity conditions for some real polynomial systems*
We study the isochronicity of centers at (0,0) for systems

$$x' = -y + A(x, y) \quad y' = x + B(x, y),$$

where A and B are real polynomials, which can be reduced to the Lienard type equation. Several new families of isochronous centers are provided. All these results are established using intensive computer algebra computations.

Tabor, Jacek (Institute of Computer Science Jagiellonian University Kraków) *Hyperbolic graph-directed IFS: definition and properties*
The notion of IFS (iterated function system) is defined for contracting mappings. In this talk we generalize this notion for hyperbolic mappings. We give a topological description of the invariant set for a graph-directed hyperbolic IFS – as a consequence we obtain estimations for its fractal dimension.

We also present how to obtain possible applications of the result for the investigation of invariant sets of discrete hyperbolic dynamical systems in $\mathbb{R}^N$.
This is joint work with Tomasz Kułaga.

Treviño, Rodrigo (The University of Maryland Department of Mathematics College Park, MD 20742 USA) *On Automated Computer-Assisted Proofs in Dynamical Systems*
Numerical and methods have been successful in uncovering chaotic behavior in dynamical systems. We present a unified approach that combines non-rigorous numerical methods with recent results in automated, rigorous computer-assisted techniques. We first find hyperbolic invariant objects whose existence we are interested in proving, then use an automated computational technique based on the discrete Conley index to find a semi-conjugacy between the given dynamical system and a symbolic dynamical system. Using the symbolics, we prove the existence of chaotic dynamics in the original system. We will illustrate these ideas with concrete examples and rigorous results.

Tucker, Warwick (Department of Mathematics Uppsala University Box 480 751 06 Uppsala Sweden) *A rigorous lower bound for the stability regions of the quadratic map*
We establish a lower bound on the measure of the set of stable parameters a for the quadratic map $Q_a(x) = ax(1-x^2)$. For these parameters, we prove that Q_a either has a single stable periodic orbit or a period-doubling bifurcation. From this result, we also obtain a non-trivial upper bound on the set of stochastic parameters for Q_a.
This is joint work with Daniel Wilczak.

Vladimirow, Vsevolod (University of Science and Technology, Krakow) *Compactons, solitons, cuspons and all that within the generalized convection-reaction-diffusion model*
I will show how the methods of normal forms and other elements of qualitative analysis can help in finding various, sometimes quite exotic, traveling wave solutions to the hyperbolic generalization of convection-reaction-diffusion equation.

Wilczak, Daniel (University of Bergen, Department of Mathematics, Johannes Brunsgate 12, 5008 Bergen, Norway.) *The C^r-Lohner algorithm and its applications*
We propose an efficient algorithm for rigorous integration of ODEs together with their variational equations up to any order [D. Wilczak, P. Zgliczyński, *C^r-Lohner algorithm*, preprint].

The main motivation for us to develop and implement it was wide spectrum of possible applications in the field of computer assisted proofs in dynamical systems. These includes

- period doubling bifurcations [D. Wilczak, P. Zgliczyński, *Period doubling in the Rössler system - a computer assisted proof*, Found. Comp. Math. (2009), published online],
- invariant tori for Poincaré maps through KAM theory [D. Wilczak, P. Zgliczyński, *C^r-Lohner algorithm*, preprint],
- homoclinic tangencies [D. Wilczak, P. Zgliczyński, *Computer assisted proof of the existence of homoclinic tangency for the Hénon map and for the forced-damped pendulum*, preprint].

In this talk I'm going to present the basic ideas of the C^r-Lohner algorithm and give some details about its application to a computer assisted proof of the existence of homoclinic tangencies. The main numerical result states that the map defined as 2π-shift along the trajectory for the forced-damped pendulum equation

$$\ddot{x} + \beta\dot{x} + \sin(x) = \cos(t)$$

admits the homoclinic tangency unfolding generically for some parameter value $\beta \approx 0.2471$. The method for verifying the existence of homoclinic tangency used in the proof requires computation of second order partial derivatives of the map.

Wilczyński, Paweł (Institute of Mathematics Jagiellonian University ul. Lojasiewicza 6/3114 30-348 Krakow Poland) *Topological entropy for local processes* We introduce a notion of topological entropy in nonautonomous continuous dynamical systems (or more precisely process) acting on a not necessarily compact space. It is a generalisation of the one introduced in [Cánovas, José. S. and Rodríguez, Jose M., *Topological entropy of maps on the real line*, Topology Appl. 153 (2005), 735–746] for nonautonomous discrete dynamical systems.

Some results was obtained in the case of periodic nonautonomous ODEs on the complex plane given in complex number notation by

$$\dot{z} = v(t, z) = (1 + e^{ikt}|z|^2)\overline{z} + f(t) + zg(t) \tag{2}$$

where $\overline{z}$ denotes the conjugate of the complex number z and f, g are treated as $\frac{2\pi}{k}$-periodic perturbations. This equation exibits a rich dynamics but all technics and notions used to describe the dynamics of (**??**) was formulated only for periodic v. It occurs that allowing v to be nonperiodic (e.g. f or g be nonperiodic) does not spoil the chaosity of dynamics. Our main goal is to investigate the relation between the chaos of the continuous system and its discretisation. We provide some necessary conditions on time sections of chaotic discretisation and the vector field of ODE which imply the chaosity of the nonautonomous continuous system. It is worth to mention that there are some topological tools e.g. method of isolating segments, which allow us to estimate the entropy or detect other types of chaos for some discretisation. These tools arises from geometric properties of the vector field and their application can be based on some rigorous computer calculations.
This is joint work with Piotr Oprocha.

Zgliczyński, Piotr (Jagiellonian University, Institute of Computer Science, ul. Lojasiewicza 6 Poland) *Periodic orbits for Kuramoto-Sivashinski PDE*