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Gianni Arioli (Politecnico di Milano)

Elliptic equations and computer assisted proofs

We presents new results concerning the existence of non-symmetric solutions and the non-existence of symmetric solutions for the equation

$$-\Delta u(z) = f\bigl(z, u(z)\bigr)\,, \quad \forall z \in \Omega\,, \qquad u(z) = 0\,, \quad \forall z \in \partial\Omega$$

when the domain Ω is either a square or a disk, and f is superlinear.
Some of this results are computer assisted. We discuss the technique used for such computer assisted proofs.

Anna Belova (Uppsala University, Sweden)

Estimation of the rotation number by interval methods

We apply interval methods to compute an accurate enclosure of the rotation number. The described algorithm is supplied with the method of proving the existence of the periodic point, which is used to check the rationality of the rotation number. A few numerical experiments were conducted to show that the implementation of interval methods produces a good enclosure of the rotation number of a circle map.

Ayse Borat (Bursa Technical University)

Higher Dimensional Motion Planners for $F(\mathbb{R}^n, k)$

Two of the main problems in Topological Robotics are to compute the topological complexity and to give a motion planner of a given space. The importance of motion planners follow not only from the fact that they give explicit motion planning algorithms but also from the fact that such algorithms can be used to compute topological complexity.

In this talk, we will introduce m-dimensional motion planners for the spaces $F(\mathbb{R}^n, k) = \{(x_1, x_2, \cdots, x_k) \in \mathbb{R}^n | x_i \neq x_j\}$. This construction of the m-dimensional motion planners tells that that $\mathrm{TC}_m(F(\mathbb{R}^n, k)) \leq m(k-1)+1$. On the other hand, regarding Theorem 1.3 in [1], this result is optimal when n is odd, but it is 1 unit away from being optimal when n is even.

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Maciej Capiński (AGH University of Science and Technology, Krakow, Poland)

Arnold diffusion in the planar elliptic restricted three-body problem

We present a diffusion mechanism for time dependent perturbations of autonomous Hamiltonian systems. The mechanism is based on shadowing of pseudo-orbits generated by two dynamics: an 'outer dynamics', given by homoclinic trajectories to a normally hyperbolic invariant manifolds, and an 'inner dynamics', given by the restriction to that manifold. On the inner dynamics the only assumption is that the system preserves area. Unlike other approaches, we do not rely on the KAM theory and/or Aubry-Mather theory; we also do not need to check twist conditions nor non-degeneracy assumptions near resonances. As an application, we study the Jupiter/Sun system restricted three-body problem. We show that for any positive and sufficiently small value of the eccentricity of the main body, there are orbits that connect the Lyapunov orbits for different energies in such a way, that the change of energy is bigger than a fixed number.

This is joint work with Marian Gidea and Rafael de la Llave.

Roberto Castelli (VU University Amsterdam)

Fourier-Taylor parameterization of invariant manifolds for periodic orbits of vector fields

Several information about the long term behaviour of nonlinear system can be inferred by studying invariant sets and connecting orbits. If the invariant sets are hyperbolic, the connecting orbits are found in the intersection of stable and unstable manifolds.

In this talk we consider periodic orbits as invariant sets and we present an efficient numerical method for computing Fourier-Taylor expansion of manifolds associated with hyperbolic periodic orbits of vector fields. The parameterisation provides covering maps for the local stable/unstable manifold with the property that they conjugate the dynamics on the invariant manifold to the dynamics of some linear system on the parameter space. Therefore the method results in accurate information about the dynamics on the manifold.

A fundamental ingredient in our construction is the Floquet theory. Indeed by continuously exploiting the Floquet normal form, the computation of the Fourier-Taylor coefficients results in solving diagonal constant coefficient algebraic equations. The technique does not require any rigorous integration and permits fast computation of the parameterisation up to any desired order also for high dimensional manifolds. Moreover the method is well suited for a subsequent rigorous validation of the computed parameterization.

This work extends the technique for the computation of the parameterization of invariant manifolds for fixed points to the case of periodic orbits and consists in a further step towards the rigorous computation of cycle-to-cycle connections.

Joint work with Jean-Philippe Lessard and Jason D. Mireles James.

Danila Cherkashin (Saint Petersburg State University)

Weak shadowing in topological dynamics

Shadowing is a very important property of dynamical systems, closely related to problems of structural stability and modelling. We look at this property from the point of topological dynamics.

Consider (X, ρ) – a compact metric space. Let a map $T : X \to X$ be continuous.

Let $\varepsilon, d > 0$. We say that $\{x_k\}_{k\in\mathbb{N}}$ is a d – *pseudotrajectory* if

$$\rho(x_{k+1}, T(x_k)) \le d$$

for all $k \in \mathbb{N}$.

We say that the mapping T satisfies *shadowing property* if for any $\varepsilon > 0$ there is a $d > 0$ such that for any d – pseudotrajectory $\{x_k\}$ there exists an exact trajectory $\{y_k = T^k(y_0), k \in \mathbb{N}\}$ such that $\rho(x_k, y_k) < \varepsilon$ for all k.

Usually this property is related to hyperbolicity. Using a completely different approach, we demonstrate that a weak version of this property may take place even for non-smooth maps. We prove following results.

Theorem 1. *For any $\varepsilon > 0$ there exists a $d > 0$ such that for any d – pseudotrajectory x_k there exists a subsequence $\{k_n\}$ and a trajectory y_k such that $\rho(x_{k_n}, y_{k_n}) < \varepsilon$.*

Let $W_0 \subset C^0(X \to X)$ be the set of all continuous maps such that for any $\varepsilon > 0$ there exists a $d > 0$ such that for any d pseudotrajectory $\{x_k\}$ there exist points $y^1, \dots, y_N$ (the number N may depend on $\{x_k\}$ and ε) such that x_k is ε close to one of points $T^k(y^i)$ for all $k \in \mathbb{N}$. Observe that for a compact smooth manifold both Anosov diffeomorphisms and identical mapping belong to W_0.

Let Q_0 be the set of all continuous maps such that for any $\varepsilon > 0$ there exists a finite ε net whose iterations are still ε nets.

Theorem 2. *$Q_0 \subset W_0$, Q_0 contains a G_δ subset of $C^0(X \to X)$. In general $W_0 \ne C^0(X \to X)$.*

For homeomorphisms, one usually consider two-sided trajectories and pseudotrajectories $\{x_k\}_{k\in\mathbb{Z}}$. Respectively, definition of shadowing changes. Similarly to Q_0 and W_0, we introduce classes Q and W of homeomorphisms of the space X.

Let Tr be the class of all transitive homeomorphisms of X and Ω be the class of homeomorphisms for which every point of X is non-wandering. Clearly, $Tr \subset \Omega$. We introduce the class M of transitive homeomorphisms such that the union of all their minimal sets is dense in X.

Theorem 3. *$Q \bigcap \Omega = W \bigcap \Omega$, $Q \bigcap Tr = M$. For any homeomorphism from the set $Q \bigcap Tr$ there exists a probability invariant measure, supported on all X.*

Luigi Chierchia (Roma 3 University)

Density of Kolmogorov tori

I shall discuss a recent result (joint with Luca Biasco) on the density of Kolmogorov tori (i.e. Lagrangian real-analytic invariant tori on which the flow is conjugated to a Diophantine linear flow) in nearly integrable Newtonian systems (Hamiltonians given by quadratic cinetic energy plus a small multi-periodic potential). Optimality and connections with numerical investigations will also be discussed.

Moshe Cohen (Technion (Israel))

The probability of choosing the unknot among 2-bridge knots using random Chebyshev billiard table diagrams

Koseleff and Pecker showed that every knot can be parametrized as a generalized harmonic curve using Chebyshev polynomials and a single phase shift. These have diagrams that appear as nice trajectories on billiard tables. We present a truncated model for random knotting using these diagrams giving all 2-bridge knots together with the unknot. We determine the probability of choosing an unknot.

This is joint work with Sundar Ram Krishnan.

Supported in part by the funding from the European Research Council under the European Union's Seventh Framework Programme, Grant FP7-ICT-318493-STREP.

Jacek Cyranka (Uniwersytet Warszawski)

A construction of two different solutions to an elliptic system

This is joint work with Piotr B. Mucha from University of Warsaw.

We construct two different solutions to an elliptic system

$$u \cdot \nabla u + (-\Delta)^m u = \lambda F$$

defined on the two dimensional torus. Here $u = (u^1, u^2)$ is sought as a vector function. The operator $(-\Delta)^m$ is elliptic homogenous of order $2m$. It can be viewed as an elliptic regularization of the stationary Burgers 2D system. A motivation to consider the above system comes from an examination of unusual propetries of the linear operator

$$\lambda \sin y \partial_x w + (-\Delta)^m w.$$

Roughly speaking the term with λ effects in a special stabilization of the norms of the operator. We shall underline that the special features of this operator were found firstly via numerical analysis. Our proof is valid for a particular force F and for $\lambda > \lambda_0$, $m > m_0$ sufficiently large. The main steps of the proof concern finite dimension approximation of the system and concentrate on analysis of features of large matrices, which resembles standard numerical analysis. Our analytical results are illustrated by numerical simulations. Experiments are agreed with the conjecture : for small m, in particular for $m = 1$ – for the classical Burgers equation with diffusion, the system does not admit solutions for large λ.

Aleksander Czechowski (Jagiellonian University)

Rigorous numerics for the FitzHugh-Nagumo slow-fast system

It is well-known that traveling waves appear in the FitzHugh-Nagumo system for sufficiently small values of the timescale separation parameter. We propose a method that allows to replace "sufficiently small" with an explicit range of the small parameter $\epsilon \in (0, \epsilon_0]$. In particular, with aid of computer, we prove the existence of a periodic solution in the traveling wave equation for ϵ_0 large enough to be reached by validated continuation procedures. In the proof we combine the methods of isolating segments and covering relations.

This is joint work with Piotr Zgliczyński

Vin de Silva (Pomona College, USA)

Topological persistence via category theory

Topological persistence originated as a strategy for measuring the topology of a statistical data set. The naive approach is to build a simplicial complex from the data and measure its homological invariants; but this approach is extremely sensitive to noise and is therefore unusable. The correct approach (made effective by Edelsbrunner, Letscher and Zomorodian in 2000) is to represent the data by a multiscale family of simplicial complexes, and to measure the homology as it varies across all scales. The resulting multiscale invariants, known as persistence diagrams, are provably robust to perturbations of the data (Cohen-Steiner, Edelsbrunner, Harer 2007).

In this talk, I will explain how these ideas may be expressed in the language of category theory. The basic concepts extend quite widely. In particular, I hope to explain how Reeb graphs and join-trees—well known constructions in data analysis—can be thought of as persistent invariants, enjoying some of the same properties as persistence diagrams. My collaborators in this work include Peter Bubenik, Jonathan Scott, Elizabeth Munch, Amit Patel.

Jarosław Duda (Uniwersytet Jagielloński)

Maximal Entropy Random Walk - when topology is not enough

Imagine the tractography problem: having a map of local diffusion tensors in a brain, we would like to reconstruct the network of its neural tracts - find the most likely network accordingly to large number of real parameters. Purely topological methods seem insufficient for this task. I will introduce to Maximal Entropy Random Walk (MERW), which is currently rapidly growing in applications, for example allowing to extract neural tracts which globally maximize the information flow. I will also tell about its other applications: for analysis of complex networks, picture analysis, optimizing informational channels, or as quantum correction for diffusion.

Marc Ethier (Uniwersytet Jagielloński)

Persistence of singular eigenspaces

Edelsbrunner, Jabłoński and Mrozek (2014) discussed how to reconstruct the dynamics of a discretely sampled map subject to noise. This involved studying the persistence of eigenspaces, for given eigenvalues over a field, for the maps in homology induced by partial simplicial maps extending the discrete map over a simplicial filtration. It required considering eigenspaces for pairs of linear maps, that is, $E_t(\varphi, \psi) = ker(\varphi - t\psi)/(ker\varphi \cap ker\psi)$ for t a field value. Doing so, they had to contend with the fact that the eigenspace for pairs can be non-trivial for every field value.

Using the classical em Kronecker canonical form, we will show how to determine and classify the eigenvalues of pairs of linear maps. For the case where all field values are eigenvalues, we will present an algorithm to extract eigenvectors parametrized by a field value t, and define the em singular eigenspace $E(\varphi, \psi)$ in order to compute their persistence. We can further establish a correspondence between these singular barcodes and those for E_t. This allows us to make sense of which eigenvalues actually represent the dynamics of the original map.

Michael Farber (Queen Mary, University of London)

Topology of Large Random Spaces

We study large random simplicial complexes (high-dimensional analogues of random graphs) and their topological and geometric properties. I will focus on a model involving several probability parameters describing the statistical properties of random complexes in various dimensions. The multi-parameter random simplicial complexes interpolate between the Linial-Meshulam random complexes and the clique complexes of random graphs. The Homological Domination Principle states that the Betti number in one specific dimension (the Critical Dimension), which depends on the probability multi-parameter, significantly dominates all other Betti numbers. Attempting to understand the general picture of properties of random simplicial complexes with a fixed critical dimension leads to a few conjectures, which I will discuss in my talk. I will also describe some results about the probabilistic treatment of the Whitehead conjecture concerning aspherical 2-dimensional complexes.

This is a joint work with A. Costa.

Jordi-Lluis Figueras Romero (Uppsala University)

How hyperbolic invariant tori bifurcate to Strange Objects: From numerics to rigorous results.

We will discuss hyperbolic invariant sets in skew-product systems of the form

$$\left\{ \begin{array}{lcl} \bar{z} & = & F(z,\theta) \\ \bar{\theta} & = & f(\theta) \text{ (mod 1)}, \end{array} \right\}.$$

where $(z, \theta) \in \mathbb{R}^n \times \mathbb{T}^d$ and f is a homeomorphism.

We will present numerics, rigorous numerics and rigorous results of how hyperbolic invariant tori bifurcate to strange invariant objects.

Peter Franek (Institute of Computer Science, Czech Republic)

Robust properties of zero sets via homotopy theory.

We study robust properties of zero sets of continuous maps $f : X \to R^n$. Formally, we analyze the family $Zr(f) = \{g^{-1}(0) : ||g - f|| < r\}$ of all zero sets of all continuous maps g closer to f than r in the max-norm. The fundamental geometric property of $Zr(f)$ is that all its zero sets lie outside of the f-preimage of a sphere of diameter r that we denote by A. We claim that once the space A is fixed, $Zr(f)$ is fully determined by an element of a certain cohomotopy group which is computable whenever the dimension of X is at most 2n - 3. By considering all r $>$ 0 simultaneously, the pointed cohomotopy groups form a persistence module – a structure leading to the persistence diagrams as in the case of persistent homology or well groups. Eventually, we get a descriptor of persistent robust properties of zero sets that has better descriptive power and better computability status than the established well diagrams. Moreover, if we endow every point of each zero set with gradients of the perturbation, the robust description of the zero sets by elements of cohomotopy groups is in some sense the best possible.

Valery Gaiko (NAS of Belarus)

Bifurcational and Topological Methods for Low-Dimensional Polynomial Dynamical Systems

We carry out the global qualitative analysis of low-dimensional polynomial dynamical systems. To control all of their limit cycle bifurcations, especially, bifurcations of multiple limit cycles, it is necessary to know the properties and combine the effects of all of their rotation parameters. It can be done by means of the development of new bifurcational and topological methods based on the well-known Weierstrass preparation theorem and the Perko planar termination principle stating that the maximal one-parameter family of multiple limit cycles terminates either at a singular point which is typically of the same multiplicity (cyclicity) or on a separatrix cycle which is also typically of the same multiplicity (cyclicity). This principle is a consequence of the principle of natural termination which was stated for higher-dimensional dynamical systems by A. Wintner who studied one-parameter families of periodic orbits of the restricted three-body problem and used Puiseux series to show that in the analytic case any one-parameter family of periodic orbits can be uniquely continued through any bifurcation except a period-doubling bifurcation. Such a bifurcation can happen, e. g., in a three-dimensional Lorenz system. But this cannot happen for planar systems. That is why the Wintner–Perko termination principle is applied for studying multiple limit cycle bifurcations of planar polynomial dynamical systems. If we do not know the cyclicity of the termination points, then, applying canonical systems with field rotation parameters, we use geometric properties of the spirals filling the interior and exterior domains of limit cycles. By means of this method, we have solved, e. g., Smale's Thirteenth Problem proving that the Liénard system with a polynomial of degree $2k + 1$ can have at most k limit cycles. Generalizing the obtained results, we have solved also the problem of the maximum number of limit cycles surrounding a singular point for an arbitrary polynomial system and Hilbert's Sixteenth Problem for a general Liénard polynomial system with an arbitrary number of singular points.

Applying a similar approach, we consider three-dimensional polynomial dynamical systems and, in particular, complete the strange attractor bifurcation scenario in the classical Lorenz system globally connecting the homoclinic, period-doubling, Andronov–Shilnikov, and period-halving bifurcations of its limit cycles. We discuss also how to apply our approach for studying global limit cycle bifurcations of discrete polynomial (and rational) dynamical systems.

This work was partially supported by the Simons Foundation of the International Mathematical Union and the Department of Mathematics and Statistics of the Missouri University of Science and Technology.

Zbigniew Galias (AGH, Kraków, Poland)

On periodic windows for the Hénon map close to the classical case

The Hénon map [1] is a two-parameter map of the plane defined by $h(x, y) = (1 + y - ax^2, bx)$. In [1], the map h is numerically studied with parameter values $(a, b) = (1.4, 0.3)$, and it is claimed that in this case "depending on the initial point, the sequence of points obtained by iteration of the mapping either diverges to infinity or tends to a strange attractor". We will refer to $(a^\star, b^\star) = (1.4, 0.3)$ as the *classical parameter values*, and we will call the attractor existing for classical parameter values the

Hénon attractor. In spite of extensive study, the long-term dynamics of the Hénon map for the classical parameter values remains unknown.

In this work, via numerical study we attempt to answer the question whether the Hénon attractor is strange/chaotic as Hénon suggested or not. To this end we develop a systematic method to search for low-period sinks in a specified region of the parameter space, and report results of applying this method to the region $Q_0 = [1.3999, 1.4001] \times [0.2999, 0.3001]$. The method is based on finding a carefully selected set of periodic orbits existing for fixed parameter values, and then using continuation in the parameter space to find a sink. This approach allows us to find many more periodic windows, and in consequence find sinks for parameter values much closer to the classical case, than using the monitoring trajectory based method [2]. We will prove that in a neighbourhood of the classical parameter values there is an abundance of periodic windows of moderately low periods. As an example, we have the following result: for
$a = 1.39999999999999999999996883990327730198456309156898 3$,
$b = 0.29999999999999999999994484551928845824433294695778 3$, the Hénon map supports a period-115 sink. These parameters satisfy the bound $\|(a, b) - (a^\star, b^\star)\|_2 < 6.335 \times 10^{-22}$.

In our systematic study, we find many (tens of thousands) low-period sinks near $(a^\star, b^\star)$. By continuation techniques, we can track the area in the parameter space occupied by the corresponding periodic windows. By a straight-forward analysis, we obtain an estimate of the probability that a point in a parameter space close to $(a^\star, b^\star)$ supports a periodic attractor: the probability that for a generic point in $Q_0 = [1.3999, 1.4001] \times [0.2999, 0.3001]$ there exist a periodic attractor is approximately 7.09×10^{-4}.

Taking into account that the majority of low-period windows in a small neighborhood of $(a^\star, b^\star)$ has been found we can estimate the probability that the Hénon attractor is periodic: the probability that a periodic attractor exists for classical parameter values $(a^\star, b^\star)$ is less than 4.59×10^{-11}.

We also study the problem of convergence to sinks existing for parameter values close to the classical case. From our analysis, it follows that even when the computations are done with sufficient precision, one may need an extremely large number of iterations to observe a sink starting from random initial conditions. The conclusion is that even if the Hénon attractor is periodic it would be practically impossible to establish this by computing trajectories of the map [3].

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Bartłomiej Garda (AGH University of Science and Technology)

An efficient method to find all low-period windows for the logistic map.

A systematic method to find the low-period windows for the logistic map $f_a(x) = ax(1-x)$ [1] will be presented. Let's define two sets $\mathcal{S}_{\text{periodic}}$ and $\mathcal{S}_{\text{chaotic}}$ of parameter values for which there exist a periodic or a chaotic attractor, respectively. It is known that these two sets are disjoint and their measures are positive and if $\mathcal{S} = [0, 4]$ denotes the whole parameter space then the measure of $\mathcal{S} \setminus (\mathcal{S}_{\text{periodic}} \cup \mathcal{S}_{\text{chaotic}})$ is zero, i.e. for almost every parameter either a chaotic or periodic attractor exist.
The goal of this work is numerical study over the set $\mathcal{S}_{\text{periodic}}$. This set can be split into an infinite number of pairwise disjoint sets $\mathcal{S}_{\text{periodic,p}}$, $p = 1, 2, \ldots$ where $\mathcal{S}_{\text{periodic,p}}$ contains parameter values for which a stable period-p orbit exists. It is known that $\mathcal{S}_{\text{periodic,p}}$ is the set build as a union of open intervals. Periodic windows can be found analytically only for very low periods. The rigorous study over the problem have been made by [2] using the bisection method. The proposed method successfully gives the estimation of the lower bound on the measure of the set $\mathcal{S}_{\text{periodic}}$.

Application of the continuation methods for finding all the periodic windows for the $p \leq 20$ [3] will be presented. For each parameter a there exists at most 2^p fixed points of f_a^p, and all of them exists (and are unstable) for $a = 4$. So the idea is to find positions of all period p orbits for $a = 4$ and then using mentioned continuation method one can find the parameter value for which the orbit becomes a sink. Using the commonly known homeomorphism conjugating the logistic map and tent-map one can find all positions of periodic orbits for $a = 4$. Then knowing the position of the periodic orbit one can set the starting points for the continuation methods. We choose the testing step size Δa and test a new point

$a_{\text{test}} = 4 - \Delta a$. Using the Newton method we check if it converges for a new point, if yes we increase the Δa and the new point is accepted ($a = a_{\text{test}}$) otherwise Δa is decreased and the test step is done again. The procedure is repeated until the Δa is smaller then some predefined threshold or a point a for which the orbit is stable is found. Then the end points of the periodic window are found. The right endpoint is found using the bisection method. And the left endpoint the continuation method is used again. If the standard double precision is used the proposed algorithm can not find all windows for $p \geq 13$. To obtain the windows of the higher periods one need to increase to computation precision. All this problems are discussed in the presentation.

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Anna Gierzkiewicz (University of Agriculture, Krakow)

Integrability of the Szekeres System

The Szekeres system is a four-dimensional system of first-order, ordinary differential equations, with nonlinear, but polynomial (quadratic) right hand side. It is an exact solution of the Einstein equations, which is inhomogeneous and has no symmetries. It has been used, among others, in modelling the early Universe or the evolution of galaxy superclusters.

I have studied the integrability of this system in the sense of finding its first integrals. The Darboux polynomial method allowed me to find two independent rational first integrals. One can obtain the last independent integral via Jacobi's last multiplier method. To find such a multiplier for Szekeres system, I have modified Goriely's method for quasimonomial systems. Therefore, the Szekeres system is completely integrable.

Javier Gomez-Serrano (Princeton University)

Computer-assisted proofs in incompressible fluids

In this talk I will discuss how, guided by numerical simulations, one can prove computer-assisted theorems in problems related to incompressible fluids using interval arithmetics. Specifically, I will talk about the Muskat (flows in a porous medium or in a Hele-Shaw cell) and the vortex patch - and its generalization known as the alpha-patch - problems. The talk is based in joint work with Angel Castro, Diego Cordoba, Rafael Granero-Belinchon and Alberto Martin Zamora.

Massimiliano Guzzo (Università di Padova)

Numerical Computation of Stable and Unstable Manifolds with Fast Lyapunov Indicators. Applications to the three body problem.

The fast Lyapunov indicators are functions defined on the tangent fiber of the phase-space of a discrete (or continuous) dynamical system. In the last decade, they have been largely used in numerical computations to localize the resonances of dynamical systems, and recently also the stable and unstable manifolds of normally hyperbolic invariant manifolds. While the traditional fast Lyapunov indicators fails to compute the manifolds in many cases, the FLI computed with a filtering window technique can be used to detect the stable-unstable manifolds with high accuracy. We illustrate the method on the critical problem of detection of the so-called tube manifolds of the Lyapunov orbits of L1, L2 in the planar circular restricted three-body problem. In particular, our three-dimensional representations of the manifolds provide an original vista of their complicate development in the phase-space.

Emmanuel Haucourt (École Polytechnique)

Directed Topology as a framework for modelling Concurrency

We describe PAML (Parallel Automata Meta Language) as an an extension of the PV language introduced by E.W. Dijkstra in 1968.

Then we build its geometric semantics explaining how the construction generalizes the usual control flow graph of sequential programs. In particular we relate the notion of d-homotopic d-paths with the notion of equivalent execution traces.

Yasuaki Hiraoka (Tohoku University)

Random Topology, Minimum Spanning Acycle, and Persistent Homology

We study a higher dimensional generalization of Frieze's $\zeta(3)$-limit theorem in the Erdös-Rényi graph process. Frieze's theorem states that the expected weight of the minimum spanning tree converges to $\zeta(3)$ as the number of vertices goes to infinity. In this talk, we study the d-Linial-Meshulam process as a model for random simplicial complexes, where $d = 1$ corresponds to the Erdös-Rényi graph process. First, we define spanning acycles as a higher dimensional analogue of spanning trees, and connect its minimum weight to persistent homology. Then, our main result shows that the expected weight of the minimum spanning acycle behaves in $O(n^{d-1})$.

Hidekazu Ito (Kanazawa University)

Integrable and Superintegrable vector fields and their normal forms at equilibria

The Poincar'e-Dulac normal form is a basic tool for analyzing behavior of solutions near an equilibrium of a given vector field. It is well known that the existence of a convergent normalization is closely related to integrability of a holomorphic vector field.

In this talk, we discuss the existence of a convergent normalization under the assumption that the vector field is "superintegrable" in such a way that the transformed system can be solved explicity. This notion of superintegrablity is a generalization of well-known superintegrable Hamiltonian systems such as Kepler problem, Eulder rigid body motion, etc.

Mateusz Juda (Jagiellonian University)

Scalable homology computing

In this talk we discuss practical approach to apply discrete Morse theory in computing homology of large data sets on multi-core, shared-memory machines. Our motivation for this work come from an application in the representation theory of unitary reflection groups, where we need to check Cohen-Macaulay property (in practice compute homology over integers) of a family of large simplicial complexes. The largest complex contains 342921600 simplices in dimension 3, so it is a difficult computational problem in terms of memory and CPU utilization.

We show a parallel algorithm for: building simplicial structure from facets and computing discrete Morse vector field. The method is more general and can be applied to any Lefschetz complex. In this particular application we get optimal number of critical cells, so we check Cohen-Macaulay property immediately. In our method we use only standard algorithms (i.e. sort, prefix sum), so it is possible to implement it in parallel computational model using well known patterns. We discuss experimental results, scalability and efficiency of the method, and tricks in the implementation.

Tomasz Kaczynski (Université de Sherbrooke)

Towards a formal tie between combinatorial and classical vector field dynamics

The Forman's discrete Morse theory is an analogy of the classical Morse theory with, so far, only informal ties. Our goal is to establish a formal tie on the level of induced dynamics. Following the Forman's 1998 paper on "Combinatorial vector fields and dynamical systems", we start with a possibly non-gradient combinatorial vector field. We construct a flow-like upper semi-continuous acyclic-valued mapping whose dynamics is equivalent to the dynamics of the Forman's combinatorial vector field, in the sense that isolated invariant sets and index pairs are in one-to-one correspondence.

This is a joint work with M. Mrozek and Th. Wanner.

Matthew Kahle (Ohio State University)

The most persistent cycles in random geometric complexes

In recent years, a number of papers have studied topological features of "random geometric complexes", particularly various facts about their expected homology. One of the motivations for this is establishing a probabilistic null hypothesis for topological data analysis. In practice, however, one usually computes persistent homology over a range of parameter, rather than homology alone. Detailed results for persistent homology of random geometric complexes have been harder to come by.

I will present new work which quantifies the length of the longest bar in persistent homology, up to a constant factor. This is an important step toward quantifying the statistical significance of topological signals in data. This is joint work with Omer Bobrowski and Primoz Skraba.

Hans Koch (The Univeristy of Texas at Austin)

On hyperbolicity in the renormalization of near-critical area-preserving maps

We consider MacKay's renormalization operator for pairs of near-critical area-preserving maps. Of particular interest is the restriction R_0 of this operator to pairs that commute and have a zero Calabi invariant. We show that a suitable extension of R_0^3 is hyperbolic at the fixed point, with a single expanding direction. Our proof is computer-assisted and yields rigorous bounds on various "universal" quantities, including the expanding eigenvalue.

Karolina Kropielnicka (University of Gdańsk, Poland)

Effective Approximation for the time dependant, linear Schrödinger equation

The computation of the semiclassical Schrödinger equation presents a number of difficult challenges because of the presence of high oscillation and the need to respect unitarity. Given periodic boundary conditions, the typical approach consists, basically, of two steps: Semi-discretisation with spectral method in space and Strang splitting in time, however this strategy occurs to be of low accuracy and sensitive to high oscillation. In this talk we sketch an alternative approach. Our analysis commences not with semi-discretisation, but with the investigation of the free Lie algebra generated by differentiation and by multiplication with the interaction potential: it turns out that this algebra possesses a structure which renders it amenable to a very effective form of asymptotic splitting: exponential splitting where consecutive terms are scaled by increasing powers of the small parameter. The semi-discretisation in deferred to the very end of computations.

Sergey Kryzhevich (Saint-Petersburg State University)

Weak shadowing in topological dynamics

Shadowing is a very important property of dynamical systems, closely related to problems of structural stability and modelling. We look at this property from the point of topological dynamics.

Consider (X, ρ) – a compact metric space. Let a map $T : X \to X$ be continuous.

Let $\varepsilon, d > 0$. We say that $\{x_k\}_{k\in\mathbb{N}}$ is a d – *pseudotrajectory* if

$$\rho(x_{k+1}, T(x_k)) \leq d$$

for all $k \in \mathbb{N}$.

We say that the mapping T satisfies *shadowing property* if for any $\varepsilon > 0$ there is a $d > 0$ such that for any d – pseudotrajectory $\{x_k\}$ there exists an exact trajectory $\{y_k = T^k(y_0), k \in \mathbb{N}\}$ such that $\rho(x_k, y_k) < \varepsilon$ for all k.

Usually this property is related to hyperbolicity. Using a completely different approach, we demonstrate that a weak version of this property may take place even for non-smooth maps. We prove following results.

Theorem 1. *For any $\varepsilon > 0$ there exists a $d > 0$ such that for any d – pseudotrajectory x_k there exists a subsequence $\{k_n\}$ and a trajectory y_k such that $\rho(x_{k_n}, y_{k_n}) < \varepsilon$.*

Let $W_0 \subset C^0(X \to X)$ be the set of all continuous maps such that for any $\varepsilon > 0$ there exists a $d > 0$ such that for any d pseudotrajectory $\{x_k\}$ there exist points $y^1, \ldots, y_N$ (the number N may depend on $\{x_k\}$ and ε) such that x_k is ε close to one of points $T^k(y^i)$ for all $k \in \mathbb{N}$. Observe that for a compact smooth manifold both Anosov diffeomorphisms and identical mapping belong to W_0.

Let Q_0 be the set of all continuous maps such that for any $\varepsilon > 0$ there exists a finite ε net whose iterations are still ε nets.

Theorem 2. *$Q_0 \subset W_0$, Q_0 contains a G_δ subset of $C^0(X \to X)$. In general $W_0 \neq C^0(X \to X)$.*

For homeomorphisms, one usually consider two-sided trajectories and pseudotrajectories $\{x_k\}_{k\in\mathbb{Z}}$. Respectively, definition of shadowing changes. Similarly to Q_0 and W_0, we introduce classes Q and W of homeomorphisms of the space X.

Let Tr be the class of all transitive homeomorphisms of X and Ω be the class of homeomorphisms for which every point of X is non-wandering. Clearly, $Tr \subset \Omega$. We introduce the class M of transitive homeomorphisms such that the union of all their minimal sets is dense in X.

Theorem 3. *$Q \bigcap \Omega = W \bigcap \Omega$, $Q \bigcap Tr = M$. For any homeomorphism from the set $Q \bigcap Tr$ there exists a probability invariant measure, supported on all X.*

Vitaliy Kurlin (Microsoft Research Cambridge, UK)

Homologically Persistent Skeleton in Computer Vision and beyond

2D images often contain irregular salient features and interest points with non-integer coordinates. Our skeletonization problem for such a noisy sparse cloud is to summarize the topology of a given 2D cloud across all scales in the form of a graph, which can be used for combining local features into a more powerful object-wide descriptor. We extend a classical Minimum Spanning Tree of a cloud to the new fundamental concept of a Homologically Persistent Skeleton, which is scale-and-rotation invariant and depends only on the given cloud without extra parameters. This graph (1) is computable in time O(n log n) for any n points in the plane; (2) has the minimum total length among all graphs that span a 2D cloud at any scale and also have most persistent 1-dimensional cycles; (3) is geometrically stable for noisy samples around planar graphs. More details are at author's website http://kurlin.org.

Claudia Landi (Università di Modena e Reggio Emilia)

Discrete Morse theory for reducing complexes in Multidimensional Persistence

The Forman's discrete Morse theory appeared to be useful for providing filtration–preserving reductions of complexes in the study of persistent homology. So far, the algorithms computing discrete Morse matchings have only been used for one–dimensional filtrations. In this talk some attempts in the direction of extending such algorithms to multidimensional filtrations are presented. Initial framework related to Morse matchings for the multidimensional setting is proposed. Moreover, matching algorithms working for 1-dimensional persistence are extended to work in the multidimensional setting. The correctness of the algorithms is proved, and its complexity analyzed. Such algorithms are used for establishing a reduction of a simplicial complex to a smaller cellular complex. First experiments with filtrations of triangular meshes are presented.

Dahisy Lima (UNICAMP - Brazil)

Smale's Cancellation Theorem: Birth and Death of Connections

Given a Morse function f on an orientable 2-manifold M, we develop an algorithm, the Smale's Cancellation Sweeping Algorithm (SCSA), that models the spectral sequence (E^*, d^*) of a filtered Morse chain complex defined by f. Whenever the SCSA identifies a non null differential d^r_p on the r-th page of (E^*, d^*), the next step of the spectral sequence produces an algebraic cancellation, i.e. $E^{r+1}_p = E^{r+1}_{p-r} = 0$.

The algebraic cancellations of the modules of the spectral sequence are dynamically interpreted as the history of birth and death of connecting orbits of the flow caused by the cancellation of consecutive critical points. Furthermore, we construct a family of gradient flows associated to the spectral sequence which also defines a continuation to the minimal flow on M.

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Alejandro Luque (Universitat de Barcelona)

Computer assisted proofs in KAM theory

In this talk we present a methodology to rigorously validate a given approximation of a quasi-periodic Lagrangian torus of a symplectic map. The approach consists in verifying the hypotheses of a-posteriori KAM theory based of the parameterization method (following Rafael de la Llave and collaborators). These hypotheses are sufficient conditions to guarantee that there exist an invariant torus with the same frequency vector close to the initial approximation.

A crucial point of our implementation is an analytic lemma that allows us to control the norm of periodic functions using their discrete Fourier transform. An outstanding consequence of this approach it that the computational cost of the validation is assymptotically equivalent of the cost of the numerical computation of invariant tori using the parametererization method.

We pretend to describe some technical aspects of our implementation and we will present some direct applications. This includes the validation of the golden invariant curve of the standard map up to a parameter value 0.9716 (that is around 0.003% of the breakdown) and validation of two dimensional invariant tori for the Froeschle map.

Irina Makarenko (Newcastle University)

3D morphology of a random field from its 2D cross-section

The two aspect ratios of randomly oriented triaxial ellipsoids (representing isosurfaces of an isotropic 3D random field) can be determined from a single 2D cross-section of their sample using the probability density function (PDF) of the filamentarity F of individual structures seen in cross-section ($F = 0$ for a circle and $F = 1$ for a line). The PDF of F has a robust form with a sharp maximum and truncation at larger F, and the most probable and maximum values of F are uniquely and simply related to the two aspect ratios of the triaxial ellipsoids. The parameters of triaxial ellipsoids of randomly distributed sizes can still be recovered from the PDF of F. This method is applicable to many shape-recognition problems, here illustrated by the neutral hydrogen density in the turbulent interstellar medium of the Milky Way. The gas distribution is shown to be filamentary with axis ratios of about 1:2:20.

Nikolai Makarenko (Russian Academy of Sciences at Pulkovo)

Geometry and topology of digital images

The use of methods of mathematical morphology, fractal geometry and computational topology to various applications is discussed.

The general context of these approaches is the idea of filtration. Excursion sets of a random field produce a nested sequence of sets forming a ring of convexity. The study of these objects led to the emergence of new areas of mathematics. Thus, the problem of Buffon's needle resulted in Crofton formulas and integral geometry. The Banach Indicatrix Function, obtained independently by Stephen Rice, formed the basis of contour statistics. The generalization of these ideas to random fields developed by Robert Adler and his team allows to use properly Jakob Steiner?s ideas and Minkowski Querschnitt Integrale to describe practical situations. Some examples will be presented during the talk.

Properties of statistical scale invariance of images, or multifractality, allow to use the estimates of smoothness or Hölder regularity for pattern analysis. Effective estimates of exponents can be obtained by means of Choquet capacities. A filtration of regularity, or multifractal decomposition of a support, allows to select components with a prescribed singularity. The most singular manifolds are useful for the analysis of textures and data compression in remote sensing.

The most popular in recent years a topological filtration allows to diagnose the spatial and temporal complexity of dynamical regimes of various processes. For chaotic dynamics, it is the only way to bring numerical solutions into agreement with analytical limitations. Some examples of computational topology applications for the solar magnetic fields, texture analysis, and brain activity will be given in the talk.

James Meiss (University of Colorado)

Using Witness Complexes to Analyze Dynamical Time Series

Standard methods for topological analysis of time-series data from dynamical systems are based on a cubical discretization and use the time series to construct an outer approximation of the underlying dynamical system. The resulting multivalued map can be used to compute the Conley index of isolated invariant sets of cubes. I will discuss a discretization that uses instead a simplicial complex constructed from a fuzzy version of the witness-landmark relationship. The goal is to obtain a natural discretization that is more tightly connected with the invariant density of the time series itself. The time-ordering of the data also directly leads to natural "witness map" on this simplicial complex. Illustrations embedded time series data from several standard dynamical models will be discussed.

Jason Mireles James (Florida Atlantic University)

Computer Assisted Proof of Saddle to Saddle Connecting Orbits for a Family of Parabolic Partial Differential Equations

An important step toward understanding the global dynamics of a nonlinear system is understanding the orbits which connect one equilibria solution to another. In this talk I will discuss some recent work on connecting orbits for parabolic partial differential equations. More specifically we are interested in proving (with the aid of the computer) the existence of connecting orbits in this infinite dimensional setting and also in studying their analytic properties. The method of proof proposed here involves several steps: computer assisted proof of the existence of equilibrium solutions of the PDE, validated study of the stability of these equilibrium solutions, numerical approximation of the stable/unstable manifolds of the equilibria, and mathematically rigorous bounds on the truncation errors associated with these approximations. Finally the connecting orbit is reformulated as the solution of a projected boundary value problem, i.e. we establish the existence of an orbit segment which begins on the unstable manifold and which arrives after some finite time on the stable manifold. Each of the sub-problems is treated using a blend of a-posteriori analysis and interval arithmetic.I will discuss how these different computer assisted proofs are linked together and also discuss our approach to studying the stable manifold (i.e. manifolds with finite co-dimension). Additional details about the Parameterization Method for unstable manifolds of parabolic PDEs and the solution of boundary value problems using Chebyshev spectral methods are discussed in the talks of Christian Reinhardt and Ray Sheombarsing.

Marian Mrozek (Jagiellonian University)

Constructing combinatorial multivector fields from data

We recall the concept of a combinatorial multivector field generalising Forman's combinatorial vector field and present an algorithm taking on input a collection of vectors on the planar integer lattice and constructing a combinatorial multivector field on the cubical CW complex of the lattice. Depending on a parameter, the algorithm may maximise the number of strict multivectors or forbid them entirely and construct a combinatorial vector field in the sense of Forman. We show examples constructed by the algorithm from a smooth planar vector field and from a collection of randomly chosen vectors. We discuss how the choice of the parameter influences the depiction of the original dynamics in the combinatorial approximation.

Sayan Mukherjee (Duke University)

Consistency of maximum likelihood estimation for some dynamical systems

We consider the asymptotic consistency of maximum likelihood parameter estimation for dynamical systems observed with noise. Under suitable conditions on the dynamical systems and the observations, we show that maximum likelihood parameter estimation is consistent. Our proof involves ideas from both information theory and dynamical systems. Furthermore, we show how some well-studied properties of dynamical systems imply the general statistical properties related to maximum likelihood estimation. Finally, we exhibit classical families of dynamical systems for which maximum likelihood estimation is consistent. Examples include shifts of finite type with Gibbs measures and Axiom A attractors with SRB measures.

Amit Patel (Institute for Advanced Study)

Persistent Homology for Maps

Suppose we have measured a map $f : X \to M$ to a Riemannian manifold M and we are interested in studying the homology of its fibers. Measurements are noisy so we do not have f but we have an approximation of f. The homology of a fiber is therefore not meaningful. What is meaningful is homology that is robust to noise.

When M is the real line, the persistent homology group offers a robust measurement. A few years back, we introduced the well group for maps to higher dimensional spaces. Since then much work has been done on both the mathematical and algorithmic sides. There are also improvements on the idea of the well group. One idea makes use of homotopy theory and another makes use of intersection theory. Finally the idea of the well group categorifies.

In this talk, I will survey the current state of this line of thought.

Alexey Petrov (St. Petersburg State University)

C^0-transversality and shadowing

The notion of a pseudotrajectory and shadowing property of a homeomorphism play an important role in the general qualitative theory of dynamical systems. From the numerical point of view, if a homeomorphism has the shadowing property, then the behavior of all numerically obtained trajectories is similar to the behavior of real trajectories.

In what follows, we denote by M a smooth closed manifold with some Riemannian metric $dist()$ and by $f\colon M \to M$ a diffeomorphism of the manifold M.

Let $d > 0$. A sequence $\xi = \{\xi_n \in M \mid n \in \mathbb{Z}\}$ is called d-pseudotrajectory for f if the following inequalities hold:

$$dist(f(\xi_n), \xi_{n+1}) \le d, \quad n \in \mathbb{Z}.$$

We say that the diffeomorphism f has shadowing property if for every $ep > 0$ there exist $d > 0$ such that any d-pseudotrajectory for f can be ep-shadowed by some point $p \in M$, i.e., the following inequalities hold:

$$dist(f^n(p), \xi_n) \le ep, \quad n \in \mathbb{Z}.$$

It is well known that for a diffeomorphism f of a closed manifold M, the following three conditions are equivalent:

(1) f has shadowing property (and this property is Lipschitz);

(2) f satisfies Axiom A and the strong transversality condition;

(3) f is structurally stable.

In [1], Sakai proved that a two-dimensional diffeomorphism with Axiom A has shadowing property if and only if it satisfies the so-called C^0-transversality condition.

In the joint paper [2] with S. Yu. Pilyugin the notion of C^0-transverse intersection of any two subsets of a topological space was introduced. Using this notion one can formulate C^0-transversality condition for any dimension (note that in the case of a two-dimensional diffeomorphism our definition coincides with the definition given in [1]).

In our talk we discuss this notion of C^0-transversality and give a sketch of construction of a three-dimensional diffeomorphism f with Axiom A that has shadowing property but fails to satisfy any transversality condition.

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Paweł Pilarczyk (IST Austria)

A combinatorial-topological approach to automatic classification of global dynamics

A computational framework will be introduced for automatic classification of global dynamics in a dynamical system depending on a few parameters. This framework is based on a set-oriented topological approach, using Conley's idea of a Morse decomposition, combined with rigorous numerics, graph algorithms, and computational algebraic topology. This method allows one to effectively compute outer estimates of all the recurrent dynamical structures encountered in the system (such as equilibria or periodic solutions), as perceived at a prescribed resolution. It thus provides a concise and comprehensive classification of all the dynamical phenomena found across the given parameter ranges. The method is mathematically rigorous, and has a potential for wide applicability thanks to mild assumptions on the system. A few specific applications in population biology, theoretical physics, and epidemiology will be highlighted.

Sergei Pilyugin (St.Petersburg State University)

Inverse shadowing for actions of finitely generated groups

The problem of shadowing for actions of some finitely generated Abelian groups was studied in [1].

In this talk, we discuss the problem of inverse shadowing for actions of some finitely generated Abelian groups. Our approach is based on the notion of “tubes" generated by continuous methods. In a sense, this notion replaces the notion of expansivity applied in [1]. medskip

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Joao Pita Costa (Institute Jozef Stefan)

The Topos Foundation of Persistence

A topos theoretic foundation for persistence allows for a common framework on the study of several aspects of persistence, illuminating the nature of concepts, establishing new theorems, suggesting more general examples, and promoting new lines of investigation. In this talk we shall look at the topos of sheaves over a Heyting algebra of lifetimes, discuss its construction and potential for a generalized simplicial homology over it. Moreover, we also describe the sheafification process through the etale space construction that permits us to compute persistent homology in the most concrete cases. Furthermore, we will discuss theorems towards stability results that can be reached at the underlying algebra level.

Alexander Prokopenya (Warsaw University of Life Sciences)

Integrable Cases of Evolutionary Equations in the Restricted Three-Body Problem with Variable Masses

The restricted three-body problem is a well-known model of celestial mechanics (see, for example, [1]). Recall that in the simplest case it describes a motion of the point P_2 of negligible mass in the gravitational field of two massive points P_0 and P_1, moving in Keplerian orbits about their common center of mass. It is assumed that the masses of points P_0 and P_1 are given and their orbits are completely determined by the known solution of the two-body problem. This problem is not integrable, and so the perturbation theory is usually applied to the analysis of the point P_2 motion. As a general solution of the two-body problem is known, one can consider in the first approximation that the point P_2 moves around the point P_0, for example, as a satellite and its Keplerian orbit is disturbed by the gravity of point P_1. Such a model has been used successfully in the study of satellite motion in the system Earth–Moon or Sun–planet [2, 3]. It was shown that doubly averaged equations of motion determining the evolution of satellite orbit may become integrable. The corresponding general solution may be found in analytic form, and it enables investigation of main qualitative features of the orbit parameters (see, for example, [4]).

We consider here a generalized case of a satellite version of the restricted three-body problem when two points P_0 and P_1 form a binary system, losing the mass due to the corpuscular and photon radiation (see [5]). We assume that the points masses vary isotropically with different rates with the only restriction that their total mass reduces according to the joint Meshcherskii law

$$\frac{m_{00}+m_{10}}{m_0(t)+m_1(t)} = \sqrt{At^2+2Bt+1} \equiv v(t) \ , \tag{1}$$

where $m_{00}=m_0(0)$, $m_{10}=m_1(0)$ are initial values of the points P_0, P_1 masses, and parameters A, B are chosen in such a way that $v(t)$ is an increasing function for $t>0$. In this case equations of the relative motion of the points P_0, P_1 are integrable and their general solution can be written in symbolic form (see [6]).

Actually, using the relative coordinates system with origin at point P_0 and assuming, without losing generality, that trajectory of the point P_1 is situated in the coordinate plane XOY, one can write Cartesian coordinates of point P_1 in the form

$$X(t)=v(t)a_1\cos\theta_1(t),\ Y(t)=v(t)a_1\sin\theta_1(t),\ Z(t)=0 \ , \tag{2}$$

where the function $\theta_1(t)$ is determined by differential equation

$$\frac{d\theta_1}{dt} \equiv \dot{\theta}_1 = \left(A-B^2+\frac{K}{a_1^3}\right)^{1/2}\frac{1}{v^2(t)} \ , \tag{3}$$

positive parameter a_1 is determined from the initial conditions, $K = G(m_{00}+m_{10})$, and G is the constant of gravitation. Note that in case of constant masses ($A = B = 0$) solution (2) determines a uniform motion of point P_1 in a circular orbit of radius a_1 with angular velocity $\omega_1 = (K/a_1^3)^{1/2}$.

Denoting position of point P_2 in the relative coordinate system by the radius-vector $\vec{R}_2$, one can write its equations of motion in the form (see [7, 8])

$$\frac{d^2\vec{R}_2}{dt^2} = -Gm_0(t)\frac{\vec{R}_2}{R_2^3} - Gm_1(t)\frac{\vec{R}_1}{R_1^3} + Gm_1(t)\frac{\vec{R}_1 - \vec{R}_2}{R_{12}^3} \ , \qquad (4)$$

where $\vec{R}_1 = (X, Y, Z)$, $R_2 = |\vec{R}_2|$, $R_{12} = |\vec{R}_1 - \vec{R}_2|$.

We analyze equation (4) under assumption that the point P_2 moves around point P_0, being perturbed by the gravity of point P_1. It is assumed also that a distance between points P_0 and P_1 is much greater than distance between points P_0 and P_2 and the Hill approximation [9] may be applied. The evolutionary equations determining long-term behaviour of the point P_2 orbital parameters are obtained by means of rewriting the equations of motion (4) in the Hamiltonian form in terms of the orbital elements and averaging them over the mean anomalies of points P_1, P_2 (see [7, 8]).

In this work we present a class of functions $m_0(t)$, $m_1(t)$, for which the evolutionary equations are integrable and describe a quasi-elliptic motion of the point P_2, and the corresponding solutions in the analytical form. Note that all relevant calculation and visualization of the results are done with the Wolfram Mathematica.

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Mariana R. da Silveira (Federal University of ABC-São Paulo-Brazil)

Continuation detected through a spectral sequence analisys.

We are interested in extracting dynamical information from algebraic topological tools found in Morse and Conley's index theory [1, 6] in order to detect continuation.

We consider a chain complex (C, Δ) generated by the critical points of a Morse function f on a compact manifold M endowed with an increasing filtration. Its differential, which is a particular case of connection matrix [6], determines an associated spectral sequence (E^r, d^r). In this context, we introduce a sweeping method [2] which produces a family of similar connection matrices and associated transition matrices [3]. The purpose of this algebraic procedure is to codify the algebraic information given by the spectral sequence in the connection matrix, in order to recover dynamical information of the initial flow and detect continuation. More specifically, we prove the the existence of certain paths in the flow associated to the nonzero differentials of the spectral sequence [2], detect bifurcation behavior in a parametrized family of flows [4] and construct a continuation to the minimal flow by associating algebraic cancellation within the spectral sequence to dynamical cancellation of critical points [5].

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Alexander D. Rahm (National University of Ireland, Galway)

A software for computations on the Dynamics and Topology of the Bianchi groups

A software for computations on the Dynamics and Topology of the Bianchi groups

Abstract. This talk is going to present the features of the software Bianchi.gp, which has been developed by the speaker for computations on the Dynamics and Topology of the Bianchi groups. The following information on the Bianchi groups can be computed with Bianchi.gp : - The orbit space of the action of the Bianchi groups on hyperbolic space, along with a symmetry-subdivided cell structure with stabilisers and identifications; - Equivariant K-homology of rings of imaginary quadratic integers; - Group homology of the Bianchi groups; - Chen-Ruan orbifold cohomology of the Bianchi orbifolds.

The Bianchi groups are defined as follows. For m a positive square-free integer, let $A(-m)$ denote the ring of algebraic integers in the imaginary quadratic field extension $Q(\sqrt{-m})$ of the rational numbers. The Bianchi groups are the projective special linear groups $PSL_2(A(-m))$.

Motivations for the study of the Bianchi groups arise in various areas of Mathematics :

- Group theory - Baum-Connes conjecture - Hyperbolic geometry - Algebraic K-theory - Knot theory - Heat kernels - Automorphic forms - Quantized orbifold cohomology.

Abhishek Rathod (Indian Institute of Science)

Min Morse: Approximability & Applications

Classical Morse theory analyzes the topology of the Riemannian manifolds by studying critical points of smooth functions defined on it. In the 90s, Forman formulated a completely combinatorial analogue of classical Morse theory now known as discrete Morse theory. In discrete Morse theory one may consider the problem of finding the number of critical simplices of the optimal discrete Morse function given an input simplicial complex $\mathcal{K}$ of size n. This problem that we refer to as the min-Morse unmatched problem (MMUP) was an open problem posed by Joswig et al. Our main result is an $\tilde{O}(n)$-time, $O(\log^2(n))$-approximation algorithm for the MMUP. Firstly, we apply an approximation preserving graph reduction procedure on MMUP to obtain a new problem namely the min-partial order problem (minPOP) - a generalization of the min-feedback arc set problem. We design two Leighton-Rao divide-&-conquer based algorithms for minPOP. The first algorithm that gives us a polynomial time $O(\log^{\frac{3}{2}} n)$ approximation involves the use of an Interior Point Method to solve the SDP followed by the use of Arora-Rao-Vazirani structure theorem to round it. The second algorithm uses Multiplicative Weights Update Method (MWUM) along with nearly linear max-flows as subroutines to give us a $O(\log^2 n)$ factor algorithm in $\tilde{O}(n)$ time. In terms of applications, as a consequence of the MWUM-based MMUP algorithm, under mild assumptions of the size of topological features being substantially small compared to the size of the complex $\mathcal{K}$, we obtain drastic reductions in time complexity for homology, persistent homology and scalar field topology (with bounded number of spurious critical cells). In the extended version of this paper, we show how one may obtain: (1.) An $\tilde{O}(n)$-algorithm for computing homology groups $H_i(\mathcal{K}, Z)$. (2) An $\tilde{O}(n^2)$-algorithm for computing persistent homology and (3.) An $\tilde{O}(n)$ algorithm for computing an approximately optimal Witten-Morse function *compatible* to input scalar function as simple consequences of the MWUM-based MMUP algorithm for giving us the best known complexity bounds for each of these applications under the aforementioned assumption.

Christian Reinhardt (VU University Amsterdam)

Rigorous computation of unstable manifolds for nonlinear parabolic PDEs via the parametrization method

We present a validated numerical method based on the so-called parametrization method to compute a parametrization of the finite dimensional local unstable manifold for a hyperbolic equilibrium solution to a scalar parabolic nonlinear PDE. As a starting point we discuss the finite dimensional analogue for hyperbolic fixed points of ODEs and explain in particular how we handle internal eigenvalue resonances occurring in the solution of the functional equation to be fulfilled by the parametrization. Next we transform the original PDE via an expansion in a suitable eigenbasis to an infinite set of ODEs. Then we discuss the analogue functional equation for the ODE posed on an infinite dimensional phase space and explain how to reformulate it as a zero finding problem for an associated nonlinear map on a suitable Banach space. This in particular can be used to efficiently compute a numerical approximate parametrization. Finally we indicate how to validate this numerical solution using a parametrized Newton Kantorovich argument. We illustrate our methods with guiding examples.

Antonio Rieser (Technion – Israel Institute of Technology)

A Topological Approach to Spectral Clustering

Topological and geometric techniques are increasingly becoming an important part of the analysis of high-dimensional, complex data sets. We present current work-in-progress of a new approach to data clustering through approximating the heat flow on a manifold. In particular, given samples from a probability distribution on a submanifold M of Euclidean space, we construct a family of approximations to the heat operator, and then use model-selection techniques in order to pick a 'topologically good' approximation to the number of connected components of M. We then use this to assign each point to one of the components to obtain a clustering of the space. We present several numerical examples, giving experimental support to the conjecture that, for a large number of points, the technique produces a correct clustering with high probability from a uniform distribution on a manifold.

Martina Scolamiero (KTH Stockholm)

Invariants for Multidimensional Persistence

Topological data analysis identifies robust shape characteristics of high dimensional data. The field has important applications ranging from sensor networks to dynamical systems and has been successfully applied to study complex biological interactions. Multidimensional Persistence, is a method in topological data analysis which allows to study several properties of a dataset at the same time. It is important to identify discrete invariants for multidimensional persistence in order to compare properties of different datasets. Furthermore such invariants should be stable, i.e data sets which are considered to be close should give close values of the invariant. This thesis addresses the problem of identifying invariants for multidimensional persistence and understanding their stability properties. In doing this, we will generalize the notion of interleaving topology on multidimensional persistence modules and consequently the notion of closeness for datasets. A filter function is usually chosen to highlight properties we want to examine from a dataset. Similarly, our new topology allows some features of datasets to be considered as noise.

Ray Sheombarsing (VU University Amsterdam)

Rigorous numerics for ODEs using Chebyshev series and domain decomposition

In this talk we present a rigorous numerical procedure for integrating polynomial vector fields by using Chebyshev series, a non-periodic analogue of Fourier series, and domain decomposition. As test problems we consider the validation of periodic and connecting orbits in the Lorenz-system.

In the first part of the talk we explain how to rigorously solve boundary value problems (BVPs) by using Chebyshev series. We start by recasting the problem under consideration into an equivalent zero-finding problem $F(x) = 0$ posed on the space of geometrically decaying sequences. The map F is then discretized, i.e. truncated to a finite dimensional map, in order to compute an approximate zero with the computer.

Next, we use a Newton-like scheme to set up a fixed-point operator T whose fixed points correspond to the zeros of F. Finally, we use a contraction argument to show that an exact zero, i.e. an exact solution of the BVP, exists in a neighborhood of the approximate zero. The latter step involves the concept of the radii-polynomials which provides an effective tool for determining a neighborhood on which the fixed-point operator is a contraction.

In the second part of the talk we discuss the use of domain decomposition. In particular, we present a heuristic procedure for computing an efficient grid for validating solutions of BVPs. The main idea is to determine a grid for which the decay-rates of the Chebyshev-coefficients on each subdomain are sufficiently high by examining the complex singularities of the orbit to be validated.

Agnieszka Siłuszyk (Siedlce University of Natural Sciences and Humanities)

New central configurations in the planar 6-body problem

In 2010, at The International Meeting on Hamiltonian systems and Celestial Mechanics, the participants stated a list of 17 open problems as a continuation of Smale's list. The problems are concerned with invariant manifolds, existence of particular solutions, singularities, relative equilibria and central configurations. The existence of central configurations is connected with two type of parameters, masses of the bodies, which are physical parameters and positions of the bodies as geometrical parameters. The main goal concerning central configurations in the n-body problem is to determine the values (m, q) of the parameters for which such configuration exists. The well known list of classical central configurations of Euler and Lagrange has been completed by B.Elmabsout [1]. He has added a configuration consisting of $2k$ equal point-masses, located at the vertices of two regular k-gons. He has proved that such configuration exists if and only these two polygons are homothetic, or differ by an angle of π/k. Now, we deal with a new family of degenerated planar twisted configuration of n-body problem, when $n = 6$. Such configuration consists of q_i, $i = 1, ..., 6$ bodies, two of them with equal masses m_1 situated at the ends of a segment having lenght 2α,while other two pairs of masses m_2 and m_3 are located at the vertices of a square whose side is $\beta\sqrt{2}$. Moreover, the vertices of this square are on the axises of symmetry of the segment. The existence of such configuration have been shown numericaly by E.A.Grebenicov (2010) [2] and have been established in the general case by A.Siluszyk (2014) [5]. We are looking firstly for necessary and sufficient conditions, imposed on masses of these bodies and on space parameters, for such central configuration to exist. Secondly, another interesting subject, which is directly connected with the degenerated central configurations is the bifurcation's problem (see E.S.G.Leandro [3], M.Sekiguchi [4]). Here, we consider a configuration of six bodies for two mass parameters $\mu = \frac{m2}{m1} > 0$ and $\nu = \frac{m3}{m1} > 0$.We deal with the following system of equations, that define the central configuration

$$U \cdot q_i = \sum_{j=1, i\neq k}^{n} m_j \frac{q_i - q_j}{r_{i\,j}^3}, \quad i = 1, ..., n \tag{1}$$

and which can be considered as a homogenous linear system with respect to the masses μ and ν, here $r_{i\,j} = |q_j - q_i|$. Taking into account our parameters we obtain from (1) the system of two equations; we will denote them by $F_1(\alpha,\beta) = F_2(\alpha,\beta) = 0$ where

$$\begin{cases} F_1(\alpha,\beta) &= f_{11}\mu + f_{12}\nu + f_{13}, \\ F_2(\alpha,\beta) &= f_{21}\mu + f_{22}\nu, \end{cases} \tag{2}$$

The coefficients $f_{11}, f_{12}, f_{13}, f_{22}, f_{23}$ are irrational and depend only on α, β; we don't discribe them here by technical reasons. In this work we study the critical values i.e. values for which the Jacobian of the system equations discribing the central configurations $\partial(F_1, F_2)/\partial(\alpha, \beta)$ is equal to zero. We will explain the bifurcation process in six bodies problem by studying the masses μ and ν.

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Primoz Skraba (Jozef Stefan Institute)

An Approximate Nerve Theorem

The nerve theorem [Borsuk 48] states that the homotopy type of a sufficiently nice topological space is captured by the nerve of a good cover of that space. Its application is crucial in computational and applied topology as it allows us to replace continuous spaces with combinatorial representation (e.g. simplicial complexes) with which we can then compute homology, cohomology, persistence, etc. of the underlying space.

In the case of persistence, we rarely compute the persistence diagram of a filtration exactly, but rather an approximation of it. In this talk we consider a notion of an epsilon-good cover and its application to computing persistence. Rather than require a good cover, one where all the elements of the cover and their finite intersections are contractible, we first replace the notion of contractible with homologically trivial and then define the notion of an epsilon good cover - one where its all elements and finite intersections are homologically trivial modulo epsilon-persistent classes (i.e. each element can have a small amount of topological noise).

We show an approximation result for the persistence diagram of a filtration of the nerve and the underlying space which depends on epsilon and the maximal dimension of the nerve and show that this bound is tight.

Kohei Soga (Keio University)

Numerical methods of weak KAM theory

In this talk, we consider finite difference approximation to weak KAM theory in the time dependent one-dimensional setting. Weak KAM theory makes clear the connection between $\mathbb{Z}^2$-periodic viscosity solutions of the Hamilton-Jacobi equation

$$\bar{v}_t(x,t) + H(x,t,c+\bar{v}_x(x,t)) = h(c)$$

and the Hamiltonian dynamics generated by H, where viscosity solutions $\bar{v}$ belong to the Lipschitz class, $c \in \mathbb{R}$ is a parameter and $h(c)$ is the effective Hamiltonian. The main objects are the graphs of $c+\bar{v}_x$, which give (backward) invariant sets, Aubry-Mather sets or KAM tori of the time-one map of the Hamiltonian system. When we think about approximation methods of weak KAM theory, it is important to approximate $\bar{v}$, $\bar{v}_x$, $h(c)$ and characteristic curves of $\bar{v}$ at the same time. This requirement makes the problem much harder, because standard frameworks of approximation to PDEs are mainly for the PDE solutions to initial value problems without any information on their derivatives or characteristic curves. Furthermore, $\mathbb{Z}^2$-periodic viscosity solutions are not unique in general for each c, which yields so-called selection problems between approximate solutions and the exact ones.

Bessi [1] shows smooth approximation methods of weak KAM theory, adding εv_{xx} to the right hand side of the Hamilton-Jacobi equation and letting $\varepsilon \to 0+$, where $\bar{v}$, $\bar{v}_x$ and $h(c)$ are smoothly approximated and the characteristic curves are approximated by stochastic processes. In this argument, the stochastic approach to viscous Hamilton-Jacobi equations introduced by Fleming [2] is very effectively exploited. Of course, the smooth approximation is not numerically available.

We consider problems similar to Bessi's through finite difference approximation. First we introduce a stochastic approach to the discretized Hamilton-Jacobi equation [3], [4]. Then we show existence of approximate objects and their convergence to $\bar{v}$, $\bar{v}_x$, $h(c)$ and characteristic curves of $\bar{v}$ [5]. In particular, we give rigorous approximation of KAM tori with error estimates depending on the Diophantine property of rotation numbers [5], and a partial answer to the selection problems in finite difference approximation [6].

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Robert Szczelina (Jagiellonian University)

Rigorous integration of Delay Differential Equations and applications

Topic of this presentation will be a method for rigorous integration of Delay Differential Equations (DDEs) that may be used to construct computer-assisted proofs of dynamical phenomena occurring in DDEs. By rigorous integration in the context of DDEs we mean a way to produce a finite representation of some set in some functional space, which contains real solutions to a given DDE. By a computer assisted proof we understand a computer program that rigorously verifies assumptions of some analytical theorems about existence of dynamical properties, e.g. Schauder fixed point theorem.

The talk will cover the theory and topological tools that link finite dimensional rigorous representations to the real solutions of DDEs, the rigorous numerical integration scheme, construction of Poincar'e Maps and an exemplary application to prove existence of a periodic solution in a highly non-linear, non-monotonous scalar DDE equation with constant delay (smooth and non-monotonous variant of the system(s) considered by Krisztin and Vas in a paper *Large-Amplitude Periodic Solutions for Differential Equations with Delayed Monotone Positive Feedback*, J. Dynam. Differential Equations 23 (2011), 727-790).

Iskander Taimanov (Sobolev Insstitute of Mathematics, Novosibirsk)

Topological analysis of three-dimesnional geological models

We discuss topological characteristics of random fields that are used for numerical simulation of oil and gas reservoirs and numerical algorithms for computing such characteristics

Katharine Turner (University of Chicago)

PCA of persistent homology rank functions with case studies in point processes, colloids and sphere packings

Persistent homology provides a method to capture topological and geometrical information in data. I will discuss the topological summary statistics called persistent homology rank functions. Under reasonable assumptions, satisfied in almost all applications, the persistent homology rank functions of interest will lie in an affine subspace. This means we can perform PCA. I will look at examples using point processes and real world data involving colloids and sphere packings. Joint work with Vanessa Robins.

Ewerton Vieira (ufg)

Transition Matrix Theory

Morse decompositions [1] and connection matrices [3] provide a supporting structure within which global bifurcations can be detected, particularly via changes in the associated algebraic structures. These differences that occur in connection matrices under continuation, which can naturally be identified algebraically, was the main motivation for the introduction of transition matrices as a combinatorial mechanism to keep track of these changes.

These transition matrices have since appeared in the literature under several guises: singular [7], topological [6], and algebraic [2]. These three types of matrices are defined differently (particularly under contrasting conditions) and have distinct properties. On the other hand, due to underlying similarities in the definitions and their corresponding properties, a unified theory for transition matrices has long been called for.

While these three types of transition matrices are each defined differently and in different settings, they have in common that each is a Conley-index based algebraic transformation that tracks changes in index information under continuation and thereby identifies global bifurcations that could occur during the continuation. It is natural to expect that the theories could be unified in an overarching transition matrix theory, and that is the main purpose of this talk, see [4] and [5].

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Hubert Wagner (IST Austria)

Topological text analysis and generalized similarity measures.

We present a new direction in topological analysis of text documents. Specifically, we focus on generalizing the notion of similarity measure from pairs of documents to arbitrary tuples. Our new definition is geometric in nature and closely related to the notion of Bregman divergence. It can be a basis for topological analysis e.g. using persistent homology. First, we present the necessary background, including computational topology (Rips and Cech complexes, persistent homology...) and text mining (vector space model, similarity measure...). Then, we contrast our specialized approach with a previous approach using standard tools such as Rips complexes. We close by discussing connections with other fields as well as future research directions.

Joint work with Herbert Edelsbrunner.

Irmina Walawska (Jagiellonian University)

BIFURCATION AND CONTINUATION OF HALO ORBITS - RIGOROUS NUMERICAL APPROACH

The Restricted Three Body Problem is a model equation for motion of a massless particle in the gravitational force of two large primaries. It attracts attention of many researches also because of its applicability to space missions, for instance the Genesis mission.

The system is given by

$$\begin{cases} \ddot{x} - 2\dot{y} = \frac{\partial\Omega(x,y,z)}{\partial x} \\ \ddot{y} + 2\dot{x} = \frac{\partial\Omega(x,y,z)}{\partial y} \\ \ddot{z} = \frac{\partial\Omega(x,y,z)}{\partial z} \end{cases} \tag{3}$$

where

$$\begin{aligned} \Omega(x,y,z) &= \frac{1}{2}(x^2+y^2) + \frac{1-\mu}{d_1} + \frac{\mu}{d_2} \\ d_1 &= \sqrt{(x+\mu)^2+y^2+z^2}, \\ d_2 &= \sqrt{(x-1+\mu)^2+y^2+z^2}. \end{aligned}$$

Our study is devoted to the analysis of an extended neighborhood for the collinear equilibrium points of The Restricted Three Body Problem. It was observed by Robert Farquhar that there is a family of symmetric, periodic orbits, parameterized by the amplitude 'z'. These orbits are called Halo orbits. Although they were found numerically, they have never been proven. We propose an algorithm for rigorous validation that the family of Halo orbits bifurcates from the family of well known planar Lyapunov orbits. We also give an algorithm for rigorous continuation of the family of Halo orbits. The method utilizes rigorous computation of higher order derivatives of well chosen Poincare map with symmetry properties of the system. As an application we give a computer assisted proof that the Halo orbits bifurcate from the family of Lyapunov orbits for wide range of the parameters μ that stand for the relative mass ratio of the two main bodies. For μ corresponding to the Sun-Jupiter system we give a proof of the existence of a wide continuous branch of Halo orbits that undergo period doubling bifurcation for some large amplitude 'z'. The computer assisted proof uses rigorous ODE solvers and algorithms for computation of Poincare maps from the CAPD library.

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Thomas Wanner (George Mason University)

Rigorous validation of isolating blocks for flows

In this talk, we present a new method for finding and rigorously verifying a special type of index pairs for finite-dimensional flows, namely isolating blocks and their exit sets. Our method makes use of a recently developed adaptive algorithm for rigorously determining the location of nodal sets of smooth functions, which combines an adaptive subdivision technique with interval arithmetic. By characterizing an exit set as a nodal domain, we are able to determine a valid index pair and proceed to compute its Conley index. Our method is illustrated using several examples for three-dimensional flows.

Anna Wasieczko-Zając (AGH Kraków)

Geometric proof of strong stable/unstable manifolds with application to the Restricted Three Body Problem.

We present a method for establishing strong stable/unstable manifolds of fixed points for maps and ODEs. The method is based on cone con- ditions, suitably formulated to allow for application in computer assisted proofs. In the case of ODEs, assumptions follow from estimates on the vector field, and it is not necessary to integrate the system. We apply our method to the restricted three body problem and show that for a given choice of the mass parameter, there exists a homoclinic orbit along matching strong stable/unstable manifolds of one of the libration points.

Frank Weilandt (Jagiellonian University, Krakow)

The discrete Conley index as the homotopy type of a space

I want to propose a new definition of the discrete Conley index. All versions of this index are local invariants for isolated invariant sets S of a time discrete dynamical system. The definition for continuous dynamical systems, i.e. flows, was developed by Charles Conley in the 1970's and is the homotopy type of a quotient space N/L. By definition, every point leaving a neighborhood N of the invariant set S, has to do so through $L \subset N$, the exit set. This definition does not depend on the choice of (N, L).

For the discrete case, a map f, one has to take some kind of equivalence class of the self map (index map) $f_{(N,L)} \colon N/L \to N/L$ induced by f. This is usually done by an equivalence relation in the homotopy category, which then leads to relations on the induced endomorphism in homology.

In this talk, we want to define the discrete Conley index as the homotopy type of a certain topological space. Apart from being easy to define and adding a geometrical flavor to the discrete Conley index theory, this space also has homology groups which can be computed from $H_*(f_{(N,L)}$ using existing homology algorithms. Even though this is still work in progress, we try to discuss the relation of this definition to the classical one given by Szymczak, Franks and Richeson via shift equivalence.

Daniel Wilczak (Jagiellonian Univeristy)

When chaos meets hyperchaos

Chaotic attractors can have a very nonuniform internal structure. Even on the plane the well known Newhouse phenomenon guarantees that small sinks of very high periods may be embedded in large chaotic zones. For higher dimensional systems one can expect coexistence of chaotic and hyperchaotic dynamics, i.e. topological horseshoes with more than one positive Lyapunov exponents.

Consider the classical 4D Rössler system

$$\dot{x} = -y - w, \quad \dot{y} = x + ay + z, \quad \dot{z} = dz + cw, \quad \dot{w} = xw + b.$$

with the parameter values $a = 0.27857$, $b = 3$, $c = -0.3$, $d = 0.05$. Let

$$\Pi = \{(x, 0, z, w) \in \mathbb{R}^3, \dot{y} = x + z < 0\}$$

be the Poincaré section and let $P : \Pi \to \Pi$ be the associated Poincaré map.

We prove that

- there is an explicitly given trapping region $B \subset \Pi$ for P, i.e. $P(B) \subset B$,

- the maximal invariant set $A = \text{inv}(P, B)$ contains three invariant sets, say H_1, H_2, H_3, on which the dynamics is Σ_2 chaotic, i.e. it is semiconjugated to the Bernoulli shift on two symbols,
- H_1 is a chaotic set with two positive Lyapunov exponents,
- H_2 and H_3 are chaotic sets with one positive Lyapunov exponent,
- there is a countable infinity of heteroclinic connections linking H_1 with H_2, H_2 with H_3 and H_1 with H_3,
- there is countable infinity of periodic orbits and heteroclinic/homoclinic orbits inside each horseshoe.

Wojciech Zakrzewski (Durham University)

Recent progress on Quasi-integrability

I this talk, based on the work done jointly with Luiz Ferreira and other collaborators, I will explain what we mean by this concept and will discuss some recent progress in this area. I plan to talk about the extra symmetries underpinning this concept and about our attempts to check whether they hold in various models invoving topological and nontopological solitons.

Krzysztof Ziemiański (University of Warsaw)

Spaces of directed paths on semi-cubical sets

One of objects used for modeling parallel computing are Higher Dimensional Automata (HDA). A special case of HDA's are PV-programs which use Dijkstra's semaphores for coordination of executions of processes. The space of all possible executions of a Higher Dimensional Automaton is the space of directed paths between two points on the underlying cubical set. I will present a construction of a CW-complex which is homotopy equivalent to such a space. This construction satisfies certain minimality condition which makes it useful for direct calculations. Furthermore, such CW-complexes carry an interesting combinatorial structure — their cells can be identified with products of permutohedra and attaching maps are inclusions of faces. I will also discuss the relationship of this construction with other models of directed path spaces.